Optimal Cost Preventative Maintenance Scheduling for High Reliability Aerospace Systems

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Abstract — Aerospace systems designed to meet stringent reliability requirements are generally expensive to replace when they fail. Preventative maintenance returning the system to a nearly new condition is often much less expensive than replacement. Selecting a cost effective preventative maintenance interval that balances the costs of replacement due to failure with the costs of periodic preventative maintenance can be challenging. The challenge is exacerbated because highly reliable aerospace systems fail infrequently, providing very little data for classical statistical analysis.

The US Coast Guard encountered such a problem with a particular subsystem on their fleet of C130 aircraft. This subsystem was the 1500 Series Flight Deck Cooling Turbine, which if failed in service cost $30,000 to replace, yet cost only $500 to overhaul as preventative maintenance. This cooling turbine had only failed five times, hardly enough data to enable use of any classical statistical method. However, this set of data was sufficient to enable conditional inferential methods to identify the preventative maintenance interval that optimizes maintenance costs at reasonable levels of risk.

This report demonstrates the parameterization of cost savings possible as a function of candidate preventative maintenance intervals for the US Coast Guard C130 1500 Series Flight Deck Cooling Turbine. Five failure data and one survivor datum are processed using conditional inferential methods. This parameterization, achieved without using any questionable assumptions, allows selection of an interval that optimizes maintenance costs. This approach may be used to select optimum cost preventative maintenance intervals for any subsystem for any aerospace system. [1] [2] [3]

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1. INTRODUCTION

The US Coast Guard fleet of C130 aircraft uses the 1500 Series Flight Deck Cooling Turbine to provide cooling and pressurization to the crew in flight. This turbine operates at 80,000+ RPM. When it fails during a mission, the crew cockpit loses cooling and pressurization. The failed turbine may emit smoke into the cockpit, produce unpleasant and raucous noise, and vibrate rather viciously. This failure interrupts the crew who must physically secure the unit. The mission is further compromised due to the subsequent limit on flight altitude for an unpressurized cockpit.

In 2002, the USCG did not perform any preventative maintenance for the 1500 Series Flight Deck Cooling Turbine, and allowed it to fail in service. Failure in service essentially damaged this system beyond repair with replacement costing $30,000. The cost for overhaul before failure in service was only $500. Faced with a 60:1 ratio of cost between replacement and overhaul, USCG decision makers identified an opportunity for immediate and significant cost savings in the maintenance of the C130 fleet.

The dilemma facing USCG decision makers in selecting the preventative maintenance interval for overhauling the 1500 Series Flight Deck Cooling Turbine was exacerbated by the fact that all C130 subsystems are designed to meet stringent reliability requirements. There existed a relative dearth of

[1] 978-1-4244-3888-4/10/$25.00 ©2010 IEEE.
[2] IEEEAC paper #1238, Version 4, Updated January 5, 2010
[3] This report is a major update to a previous work [1]. This report provides a more complete analytical development, corrects a number of inadequacies and misnomers, and adds a discussion of application of the required numerical methods enabling readers of this report to not only duplicate the results, but apply them to their own aerospace system problems.

failure data for the 1500 Series Flight Deck Cooling Turbine. USCG decision makers, as typical of all engineering decision makers, are comfortable making decisions when hundreds to thousands of data are available. As the numbers of data decrease, so does the decision maker's comfort with making the decision. With very few data, a decision may be postponed if not outright abandoned. The USCG had only five failure data, and faced this uncomfortable decision directly. Postponement of this decision produces missed cost savings, yet selecting a preventative maintenance interval that did not offer the minimum risk for maximum cost savings could be career limiting. This is a common problem in management of modern aerospace systems and subsystems.

This report outlines a procedure to enable selection of a preventative maintenance interval that optimizes maintenance costs where few failure data are available. The procedure uses conditional inferential methods and does not require use of questionable assumptions. Using five failure data and one survivor datum for the USCG C130 1500 Series Flight Deck Cooling Turbine, this report exercises this procedure to demonstrate how maintenance cost risk is parameterized as a function of candidate preventative maintenance intervals. Optimal cost preventative maintenance scheduling can be developed for high reliability aerospace systems where few data are available.

2. METHODS

Failure Model Selection

Selection of a model for the uncertainty about the data is a task based on knowledge of the physics that produces the data and the basic characteristics of the data. The data to be analyzed for reliability related problems consists of times of failure, and times at which the unit was observed to have not failed as yet, commonly called survivors or suspensions. These data are one-sided; they can only have positive semi-definite values. Further, for the USCG C130 1500 Series Flight Deck Cooling Turbine, there is no reason to suspect that the physics involved in failure would produce a multi-modal time of failure model. Beyond these two facts, nothing more can be presumed about the distribution of failure times.

With those considerations, an uncertainty model suitable for these data would be the Weibull model [2] [3]. The Weibull model is the most general one-sided uni-modal uncertainty model available. The Weibull model can provide a variety of distribution shapes, singular at zero time, left-skewed, right-skewed, and nearly symmetric as illustrated in Figure 1.

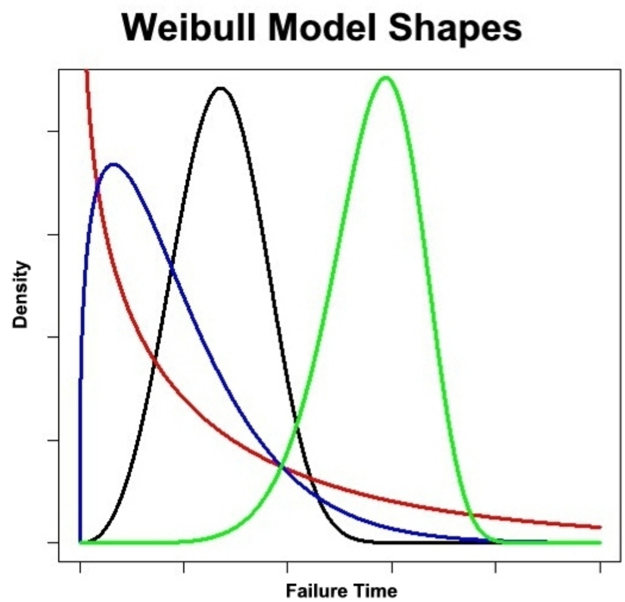

Figure 1 — Being the most general of the one-sided uni-modal uncertainty models, the Weibull model provides a wide variety of shapes for failure time models.

Equation (1) provides the general Weibull density function, which has a location parameter t_1, a scale parameter η, and a shape parameter β.

$$pd(t_f \mid t_1, \eta, \beta) = \left(\frac{\beta}{\eta}\right)\left(\frac{t_f - t_1}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_f - t_1}{\eta}\right)^{\beta}} \quad (1)$$

The Weibull model is a rather nice model for reliability related problems in that the parameters all have physical meanings. This is not the case for many probability distribution models. The location parameter t_1 represents the time before which failures cannot occur, and is called the failure-free time. The scale parameter η is the time at which 63.2% of all failures will have occurred, and is called the critical life. The shape parameter β is an indicator of failure mode. Values of $\beta < 1$ indicate an infant mortality failure mode. Values of $\beta = 1$ indicate a useful life failure mode. Values $1 < \beta < 4$ indicate an early wearout failure mode. And, values of $\beta > 4$ indicate an old age failure mode.

An important aside relative to this density formulation: Weibull's original paper [2] published in September 1951 provided a distribution function that would produce the density function in equation (2).

$$pd(t_f \mid t_1, \lambda, \alpha) = \left(\frac{\alpha}{\lambda}\right)\left(t_f - t_1\right)^{\alpha-1} e^{-\frac{\left(t_f - t_1\right)^{\alpha}}{\lambda}} \quad (2)$$

In discussions of Weibull's paper [3] published in June 1952, Weibull noted that his distribution function as originally published was incorrect by stating that the

"…parentheses are an awkward misprint." Correction of this misprint produces the density function in equation (1).

The significance of this typographical error is profound. First, equation (2) cannot be reparameterized to produce equation (1); the density function in equation (2) is fundamentally flawed since neither λ nor α can be classed as proper location, scale, or shape parameters. Second, textbooks [4] [5] exist that use the incorrect density function in equation (2) for the Weibull model. And, third, there are statistical software packages and tools [6] [7] that use the incorrect density function in equation (2) for the Weibull model. The caveat for the reader of this report is that whenever encountering any work using the Weibull model, and when considering any software package or tool, it is imperative to verify that the implementation of the Weibull model uses a form expressible as equation (1). The results obtained in any analytical work or through use of a software package that uses a form expressible as equation (2) will be pathological.

For the work presented in this report, the location parameter t_1 in equation (1) is set to zero. There exists no reason to believe that any cooling turbine could not fail the instant operation begins.

Conditional Inferential Approach

The first step in determining an optimal cost preventative maintenance interval for an aerospace subsystem is to infer from the data the uncertainty model for the parameters of the selected failure model, for the Weibull distribution model. With conditional inferential methods, the joint probability density model for the parameters of the Weibull distribution is developed based solely on the data. With this joint density, it is possible to compute any probability calculation that might be useful. To develop the joint density of η and β given the data, Bayes' Law [8] is employed per equation (3).

$$pd\left(\eta,\beta \mid data\right) \propto pd\left(data \mid \eta,\beta\right) pd\left(\eta,\beta\right) \quad (3)$$

In equation (3),the first term to the right of the proportion, $pd(data|\eta,\beta)$, is the likelihood. When the data is limited to only failure times, this is the same likelihood function used in calculating maximum likelihood estimates. The second term to the right of the proportion, $pd(\eta,\beta)$, is the joint prior density for η and β. The joint prior density is selected to model the knowledge or ignorance of η and β before obtaining the data. The proportionality in the equation is insignificant; the proportionality constant can always be calculated by integrating over all values of η and β. The term on the left, $pd(\eta,\beta|data)$, the joint density of η and β given the data, is called the joint posterior density.

Selection of the prior model for some problems can pose some difficulty. Some decision makers feel that using a priori knowledge of the parameters somehow prejudices the results, casting the pall of a rigged decision subject to second-guesses. Beyond that, for many uncertainty models that might be selected for the data for various problems, the parameters have no useful physical meaning, and thus no reason exists to have any a priori knowledge of them. To address both of these difficulties, it is possible to use a prior density model that imparts no a priori knowledge of the parameters. This is called using a noninformative or ignorance prior [9]. Use of ignorance priors establishes a basis of maximum objectivity for the decision, and alleviates the difficulty of dealing with any second-guessing. The joint prior density model is generally structured such that the parameters are independent. Using the Weibull model, because η and β are scale and shape parameters respectively, Jeffrey's priors [10] are very suitable as the ignorance priors for η and β and are presented in equations (4).

$$pd\left(\eta\right) \propto \frac{1}{\eta}; \quad pd\left(\beta\right) \propto \frac{1}{\beta} \qquad (4)$$

Now, given as data N_f failures and N_s survivors (times of good inspections or when some other unrelated failure occurred – e.g., a maintenance mechanic breaks off a stud in inspection), the posterior density model is formed in equation (5) using the Weibull distribution from equation (3) with t_{fi} being the i^{th} failure time, and t_{sj} being the j^{th} survivor time.

$$pd\left(\eta,\beta \mid data\right) \propto \left(\prod_{i=1}^{N_f} \left(\frac{\beta}{\eta}\right)\left(\frac{t_{fi}}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_{fi}}{\eta}\right)^{\beta}} \right) * \left(\prod_{j=1}^{N_s} e^{-\left(\frac{t_{sj}}{\eta}\right)^{\beta}} \right) * \left(\frac{1}{\eta}\right) * \left(\frac{1}{\beta}\right) \qquad (5)$$

In equation (5), the first term to the right of the proportion is the likelihood for the failure data, the second is the likelihood for the survivor data, and the two remaining terms are the Jeffrey's priors for η and β. One very nice feature of conditional inferential methods apparent in equation (5) is that survivor data can be used directly via the likelihood [11]. Observed times at which a subsystem in service has not failed comprise very important information that should not be neglected in the posterior or in the decision. For some problems, the number of survivor data may exceed that for failure data, and there may be only survivor data and no failure data at all. Conditional inferential methods provide solutions for these data sets; such solutions are not possible using classical methods without employing assumptions that may be questionable.

Development of Figures of Merit

There are three Figures of Merit (FOM's) useful to a decision maker faced with selecting a preventative maintenance interval for a subsystem with a known cost to replace, C_{rep}, and a known cost of preventative maintenance, C_{pm}. The first is C_{fis}, the maintenance cost per flight hour for replacing subsystems that fail in service. The second is C_{tpm} the cost per flight hour for the subsystem when providing periodic preventative maintenance at intervals of t_{pm} in flight hours. The third and most important to the decision maker is CS_{tpm}, the cost savings per flight hour when providing preventative maintenance at intervals of t_{pm}, over the current maintenance procedure of replacing subsystems that fail in service. All of these FOM's are uncertain, and inferring the uncertainty in them based on the data allows computation of the cost risk in selecting a preventative maintenance interval.

Since the current procedure allows the subsystem units to fail in service, the cost of maintenance per flight hour is not incurred until the unit fails. The uncertainty in C_{fis} for the entire fleet is inherent in the uncertainty in failure time for all units, which is modeled by the Weibull model in equation (1) with t_1 set to zero. The uncertainty in C_{fis} then becomes a function of the failure time and the values of the parameters of the Weibull model as shown in equation (6).

$$pd(C_{fis} \mid t_f, \eta, \beta) \propto \left(\frac{C_{rep}}{t_f} \right) pd\left(t_f \mid \eta, \beta \right) \quad (6)$$

If the values of η and β were known exactly, this equation could be integrated to calculate the probability that the cost of the current procedure were above or below some selected value. Unfortunately, η and β are uncertain as well. Fortunately, the uncertainty model for η and β can be inferred solely from the data using the conditional inferential approach presented in this report. Equation (7) shows how the uncertainty model including the uncertainty in the data is developed hierarchically.

$$\begin{aligned} &pd(C_{fis} \mid t_f, \eta, \beta, data) \\ &\quad \propto pd(C_{fis} \mid t_f, \eta, \beta) pd\left(\eta, \beta \mid data \right) \end{aligned} \quad (7)$$

For this decision, uncertainties about the time of failure and the values of the parameters of the failure time uncertainty model are essentially irrelevant. The decision maker wants to know the uncertainty for C_{fis} based on the actual data. It is not necessary to consider a range of hypothetical values for time of failure and parameters for the failure uncertainty model. The uncertainty model for C_{fis} based solely on the data is obtained by marginalizing the uncertainty model expressed in equation (7) with respect to the uninteresting terms. Marginalization eliminates uninteresting terms by integrating them out as demonstrated in equation (8).

$$\begin{aligned} &pd(C_{fis} \mid data) \\ &\quad \propto \int_0^\infty \int_0^\infty \int_0^\infty pd(C_{fis} \mid t_f, \eta, \beta, data) dt_f \, d\eta \, d\beta \end{aligned} \quad (8)$$

Note in the left side of equation (8) that t_f, η, and β no longer appear, providing the uncertainty model for C_{fis} based solely on the data. With the substitution from equation (1) into equations (6), (7) and (8), and substitutions of equations (6) and (7) into equation (8), the innermost integral in equation (8) has an analytical solution.

$$\begin{aligned} &\int_0^\infty \left(\frac{C_{rep}}{t_f} \right) \left(\frac{\beta}{\eta} \right) \left(\frac{t_f}{\eta} \right)^{\beta - 1} e^{-\left(\frac{t_f}{\eta} \right)^\beta} dt_f \\ &\quad = \left(\frac{C_{rep}}{\eta} \right) * \Gamma \left(\frac{\beta - 1}{\beta} \right) \end{aligned} \quad (9)$$

The result then is that a double marginalization integral remains.

$$\begin{aligned} &pd(C_{fis} \mid data) \\ &\quad \propto \int_0^\infty \int_0^\infty \left(\left[\left(\frac{C_{rep}}{\eta} \right) * \Gamma \left(\frac{\beta - 1}{\beta} \right) \right] \right. \\ &\qquad \left. * pd\left(\eta, \beta \mid data \right) \right) d\eta \, d\beta \end{aligned} \quad (10)$$

C_{tpm} based on the data is developed similarly. Clearly, the cost per flight hour for units that do not fail is simply the cost for preventative maintenance (C_{pm}) divided by the preventative maintenance interval time (t_{pm}). Uncertainty for failure time applies for C_{tpm} very much as it did in equations (6) and (7), but with a slight twist.

$$\begin{aligned} &pd(C_{tpm} \mid t_f, \eta, \beta, data) \\ &\quad \propto \left(\frac{C_{rep}}{t_f} \right) pd\left(t_f \mid \eta, \beta \right) pd\left(\eta, \beta \mid data \right) \\ &\text{for } t_f < t_{pm} \\ &\quad \propto \left(\frac{C_{pm}}{t_{pm}} \right) pd\left(\eta, \beta \mid data \right) \\ &\text{for } t_f \geq t_{pm} \end{aligned} \quad (11)$$

Again, uncertainties about the time of failure and the values of the parameters of the failure time uncertainty model are irrelevant. The key for C_{tpm} is to know the uncertainty model based on the data. Marginalization may be used as it was in equations (8) and (10) with the inner most integral being split between periods before t_{pm} and after.

$$pd(C_{tpm} \mid data) \propto \int_0^\infty \int_0^\infty \left[\int_0^{t_{pm}} \left(\frac{C_{rep}}{t_f} \right) pd\left(t_f \mid \eta, \beta\right) dt_f + \int_{t_{pm}}^\infty \left(\frac{C_{pm}}{t_{pm}} \right) pd\left(t_f \mid \eta, \beta\right) dt_f \right] * pd\left(\eta, \beta \mid data\right) d\eta d\beta \quad (12)$$

Both of the inner most integrals in equation (12) have analytical solutions after substitution of equation (1).

$$\int_0^{t_{pm}} \left(\frac{C_{rep}}{t_f} \right) \left(\frac{\beta}{\eta} \right) \left(\frac{t_f}{\eta} \right)^{\beta-1} e^{-\left(\frac{t_f}{\eta}\right)^\beta} dt_f = \left(\frac{C_{rep}}{\eta} \right) \left[\Gamma\left(\frac{\beta-1}{\beta} \right) - \gamma\left(\frac{\beta-1}{\beta}, \left(\frac{t_{pm}}{\eta} \right)^\beta \right) \right] \quad (13)$$

where $\gamma(\bullet,\bullet)$ is the upper incomplete Gamma function.

$$\int_{t_{pm}}^\infty \left(\frac{C_{pm}}{t_{pm}} \right) \left(\frac{\beta}{\eta} \right) \left(\frac{t_f}{\eta} \right)^{\beta-1} e^{-\left(\frac{t_f}{\eta}\right)^\beta} dt_f = \left(\frac{C_{pm}}{t_{pm}} \right) * e^{-\left(\frac{t_{pm}}{\eta}\right)^\beta} \quad (14)$$

Substitution of equations (13) and (14) into equation (12) produces the final marginalization equation required.

$$pd(C_{tpm} \mid data) \propto \int_0^\infty \int_0^\infty \left[\left(\frac{C_{rep}}{\eta} \right) * \left\{ \Gamma\left(\frac{\beta-1}{\beta} \right) - \gamma\left(\frac{\beta-1}{\beta}, \left(\frac{t_{pm}}{\eta} \right)^\beta \right) \right\} + \left(\frac{C_{pm}}{t_{pm}} \right) * e^{-\left(\frac{t_{pm}}{\eta}\right)^\beta} \right] * pd\left(\eta, \beta \mid data\right) d\eta d\beta$$

(15)

CS_{tpm} is also subject to uncertainty based on the data, and is developed using the results for C_{tpm} and C_{fis}. When a failure occurs before the preventative maintenance interval ($t_f < t_{pm}$), there is no cost savings. When a unit survives to t_{pm} such that preventative maintenance is performed, there is a potential cost savings that depends on the value of t_{pm}. If t_{pm} is selected to be very small, there could be additional costs per flight hour incurred. If t_{pm} is selected to be very large, all units will fail before preventative maintenance and there will be no cost savings. For CS_{tpm} given the data, the inner most integral will be split similarly at t_{pm} as in equation (12). The integrand for the integral for the period before t_{pm} consists of the integrand from equation (10) from which is subtracted the inner most integral for the period before t_{pm} from equation (12). Clearly, the inner most integral for the period before t_{pm} for CS_{tpm} is identically zero. The integrand for the integral for the period after t_{pm} consists of the integrand from equation (10) from which is subtracted the inner most integral for the period after t_{pm} from equation (12). This remaining inner most integral for the period after t_{pm} has an analytical solution and provides the marginalization integral in equation (16) for CS_{tpm} given the data.

$$pd(CS_{tpm} \mid data) \propto \int_0^\infty \int_0^\infty \left[\left(\frac{C_{rep}}{\eta} \right) \gamma\left(\frac{\beta-1}{\beta}, \left(\frac{t_{pm}}{\eta} \right)^\beta \right) - \left(\frac{C_{pm}}{t_{pm}} \right) * e^{-\left(\frac{t_{pm}}{\eta}\right)^\beta} \right] * pd\left(\eta, \beta \mid data\right) d\eta d\beta \quad (16)$$

Numerical Methods

To calculate the cost risk in selection of a preventative maintenance interval, it is necessary to obtain the uncertainty models for the FOM's given the data. This requires evaluation of equations (10), (15), and (16). None of these equations are analytically integrable. The solution is to use numerical methods, namely Monte Carlo methods [12]. Monte Carlo methods are used widely for accurately approximating the evaluation of probability integrals. Quite often, risk problems such as the subject of this report are solvable only using Monte Carlo methods.

The central issue to evaluating equations (10), (15), and (16) using Monte Carlo methods is to obtain a large number of samples of η and β from the joint posterior uncertainty model in equation (5). There exist no statistical software packages with built-in samplers for the joint posterior density function of equation (5). The remedy is to use Markov Chain Monte Carlo (MCMC) methods to sample this posterior. MCMC methods allow full range sampling of arbitrary distributions of any dimension given the formulation of the joint density [13]. With sufficient

MCMC sampling of the joint posterior in equation (5), it is possible to compute very accurate approximations for almost any measure or statistic of interest, including evaluations of equations (10), (15), and (16).

Once the joint MCMC samples of η and β are obtained, the integrals are evaluated for equations (10), (15), and (16) using a non-intuitive yet simple process. Monte Carlo samples of the uncertainty models for C_{fis}, C_{tpm} , and CS_{tpm} may be obtained by simply evaluating the central integrands in equations (10), (15), and (16) at these joint MCMC samples of η and β.

There remains one difficulty with this process, and it appears in all three equations (10), (15), and (16). First, the upper incomplete Gamma function is not defined when the first argument is less than zero. Clearly in equations (15) and (16), if the MCMC sample of β is less than unity, then this first argument for the upper incomplete Gamma function will be less than zero. To investigate this pathology, sets of ordinary Monte Carlo samples were obtained for failures using the Weibull uncertainty model with a given value of η and a range of values $0 < \beta \le 5$. Thus, the numerical evaluation of the integral in equation (13) was compared with the analytical evaluation over this range of values for β. Using 5,000 Weibull failure samples for each value of β, agreement was obtained between the numerical and analytical solution to at least three decimal places for any β>1.5, with divergence beginning below β=1.5, and with significant divergence for any β< 1.2. The Monte Carlo results were nicely continuous over the full range of β to almost zero.

Another related pathology appears in equations (10) and (15) for the Gamma function evaluations. If the MCMC sample of $\beta =1/n$ for all positive integers n = 1, 2, 3, … , ∞, then the Gamma function argument will be $1 - n$ respectively, each of which results in a complex pole, another major problem in equations (10) and (15). In between these Gamma function argument values, the Gamma function alternates between positive and negative values, which in the absolute sense in these regions can get very close to zero. Neither the complex poles nor the negative values are realistic, since for infant mortality modes when β< 1, failures times will be small, approaching zero, causing C_{fis} and C_{tpm} to approach +∞. This analytical pathology was also investigated by comparing with Monte Carlo results. Again, three digits of agreement between analytical and Monte Carlo solutions were obtained for values of β >1.5, with divergence below β=1.5. As expected intuitively, positive Monte Carlo results were obtained for all values of β, and as $\beta \to 0$, the Monte Carlo values for C_{fis} and $C_{tpm} \to +\infty$. The author welcomes from readers of this report any explanation for this observed pathology in the analytical formulation for β<1.5.

To avoid these pathologies in calculating samples of the uncertainty models for the FOM's using the joint MCMC samples of η and β, whenever a sample β <1.8 occurs, a large number of samples from the Weibull failure time uncertainty model for those joint values of η and β will be used in a Monte Carlo evaluation of the inner integrals, instead of the analytical formulae. For C_{fis}, the inner integral value is approximated using a set of N (large, at least 10,000) Monte Carlo Weibull failure time samples T_f for each value of η and β via the formula in equation (17).

$$\int_0^{\infty}\left(\frac{C_{rep}}{t_f}\right)\left(\frac{\beta}{\eta}\right)\left(\frac{t_f}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_f}{\eta}\right)^{\beta}} dt_f \cong \frac{\sum_{i=1}^{N}\left(\frac{C_{rep}}{T_{fi}}\right)}{N} \qquad (17)$$

For C_{tpm}, the inner integral value was approximated using a set of N (large, at least 10,000) Monte Carlo Weibull failure time samples T_f for each value of η and β using the formula in equation (18).

$$\int_0^{t_{pm}}\left(\frac{C_{rep}}{t_f}\right)\left(\frac{\beta}{\eta}\right)\left(\frac{t_f}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_f}{\eta}\right)^{\beta}} dt_f + \int_{t_{pm}}^{\infty}\left(\frac{C_{pm}}{t_{pm}}\right)\left(\frac{\beta}{\eta}\right)\left(\frac{t_f}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_f}{\eta}\right)^{\beta}} dt_f \cong \frac{\sum_{all\,T_f<t_{pm}}\left(\frac{C_{rep}}{T_f}\right)+\sum_{all\,T_f\ge t_{pm}}\left(\frac{C_{pm}}{t_{pm}}\right)}{N} \qquad (18)$$

For CS_{tpm}, the inner integral value was approximated for a set of N (large, at least 10,000) Monte Carlo Weibull failure time samples T_f for each value of η and β using the formula in equation (19).

$$\int_{t_{pm}}^{\infty}\left[\left(\frac{C_{rep}}{T_f}-\frac{C_{pm}}{t_{pm}}\right)\right]\left(\frac{\beta}{\eta}\right)\left(\frac{t_f}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_f}{\eta}\right)^{\beta}} dt_f \cong \frac{\sum_{all\,T_f\ge t_{pm}}\left(\frac{C_{rep}}{T_f}-\frac{C_{pm}}{t_{pm}}\right)}{N} \qquad (19)$$

Note that the left side of equation (19) does not explicitly account for the zero cost savings for failure times $T_f < t_{pm}$, but the right side does account for them.

As a note, this Monte Carlo evaluation process for the inner integrals for C_{fis}, C_{tpm} , and CS_{tpm} could be used for all values of β using equations (17), (18), and (19), completely ignoring the analytical solutions. This of course increases by orders of magnitude the computation time required to obtain the samples of the uncertainty models given the data for C_{fis}, C_{tpm}, and CS_{tpm}, and perhaps introduces small numerical method errors.

With the samples of C_{fis}, C_{tpm}, and CS_{tpm}, it is now very simple to calculate cost risks. For example, to compute the risk based on the data that the cost savings per flight hour will be less than \$5 for t_{pm} =100 hours, all that is necessary is to count the number of samples of CS_{tpm} < \$5 for t_{pm} =100 hours and divide by the total number of samples.

3. RESULTS

Failure and Survivor Data

The data provided by the USCG for the C130 1500 Series Flight Deck Cooling Turbine consisted of five failures requiring replacement and one survivor. The failures were at 463, 538, 1652, 1673, and 2462 flight hours, and the survivor at 96 flight hours. No information was provided on the failure modes inherent in these data or on the details of the overhaul procedure for the survivor. The USCG fleet of C130 aircraft numbers close to 130. No information was provided on how many C130 aircraft in this fleet are equipped with the 1500 Series Flight Deck Cooling Turbine, beyond the five which experienced failures. If more C130 aircraft than these five were equipped with the 1500 Series Flight Deck Cooling Turbine, the survivor data for these additional aircraft was not provided. The results presented in this report may be somewhat more conservative than if these absent survivor data existed and had been provided. As provided by the USCG, the replacement cost, C_{rep}, was \$30,000, and the overhaul cost, C_{pm}, was \$500. The information/data set in toto as provided by the USCG for this problem introduces many new questions, the answers to which only can improve the cost results of the analysis presented in this report. However, this limited data set is more than sufficient to demonstrate the advantages of using these methods for highly reliable aerospace systems.

Generally, five failure data are not enough to allow a USCG decision maker to comfortably select an overhaul interval with such a large replacement/overhaul cost ratio.

Monte Carlo Sampling

The conditional inferential procedure described in this report was used to process the five failure and one survivor data using a Metropolis-Hastings sampling algorithm [14] to obtain 10,000 MCMC joint posterior samples of η and β. Figure 2 presents these samples, and the density of the points reflects the joint density of η and β given the data.

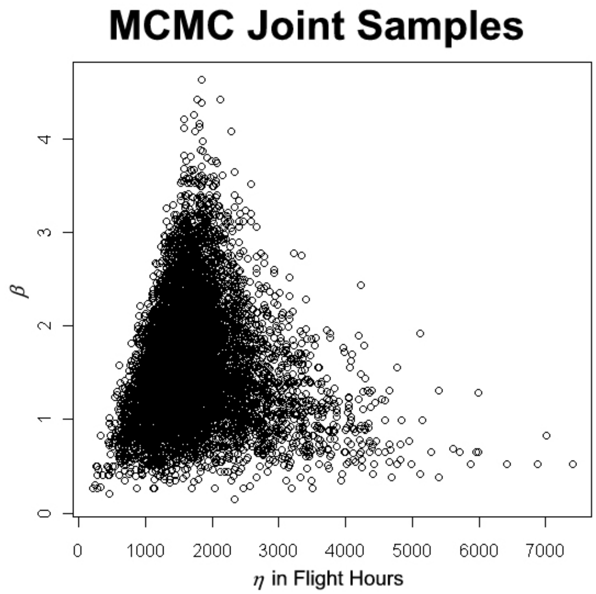

Figure 2 — The 10,000 joint samples of η and β obtained using a Markov Chain Monte Carlo process based on the five failure and one survivor data show a non-intuitive joint distribution.

Figure 2 is interesting since it displays a joint distribution that looks almost like a pork chop, something no known built-in sampler for any statistical tool will produce. For such problems using conditional inferential methods, it is always interesting and entertaining, yet rarely intuitive, to observe the joint models that result from differing data sets.

Costs with No Preventative Maintenance

Using the samples of η and β given the data in Figure 2, samples of C_{fis}, the maintenance cost per flight hour of allowing the C130 1500 Series Flight Deck Cooling Turbine to fail in service, are computed evaluating equation (10) using the procedure discussed in the section of this report on numerical methods. The uncertainty model based on the data for C_{fis} is displayed in Figure 3.

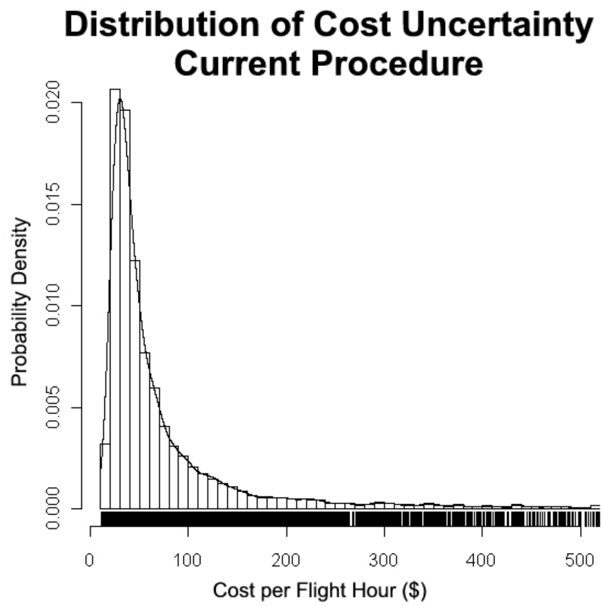

Figure 3 — The uncertainty distribution for C_{fis}, the cost of maintenance per flight hour allowing the turbine to fail in service, shows appreciable risk at very large cost per flight hour values.

Using the uncertainty model contained in the samples of C_{fis}, the risk based on the data that the current procedure exceeds a specified dollar value per flight hour can be computed directly. Figure 4 displays these risks parameterized as a function of costs per flight hour.

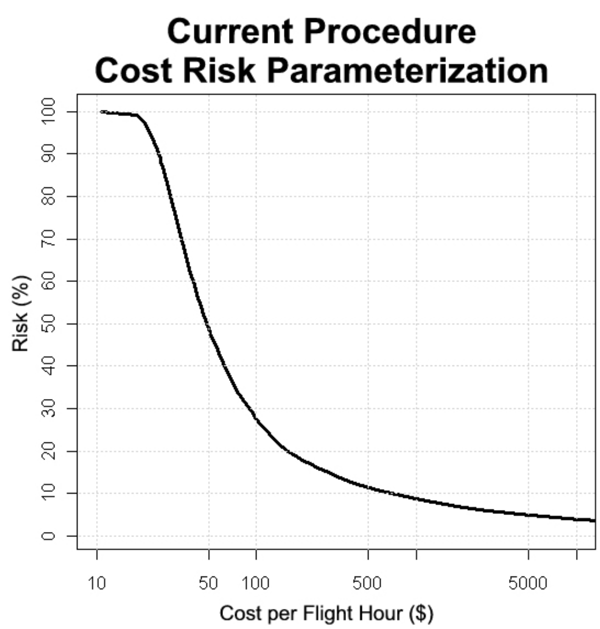

Figure 4 — Based on the available data, there is a 50% risk that maintenance cost will exceed $49 per flight hour when the turbine is allowed to fail in service.

As can be seen in Figure 4, the current procedure of allowing the turbine to fail in service is essentially guaranteed to exceed $10 per flight hour based on the data. Figure 4 shows some other important results as well; based on the data, the cost of the current procedure has a 10% risk of exceeding $698 per flight hour.

Costs Parameterized by Maintenance Interval

Using the samples of η and β given the data in Figure 2, samples of C_{tpm}, the maintenance cost per flight hour for preventative maintenance of the C130 1500 Series Flight Deck Cooling Turbine can be parameterized as a function of preventative maintenance interval t_{pm} by evaluating equation (15) and using the procedure discussed in the section of this report on numerical methods. Figure 5 displays the cost per flight hour of preventative maintenance, based on the data, parameterized as a function of t_{pm}, at four different risk levels (95% – red line, 75% – orange line, 50% – green line, and 25% – blue line) of exceeding the cost per flight hour on the ordinate.

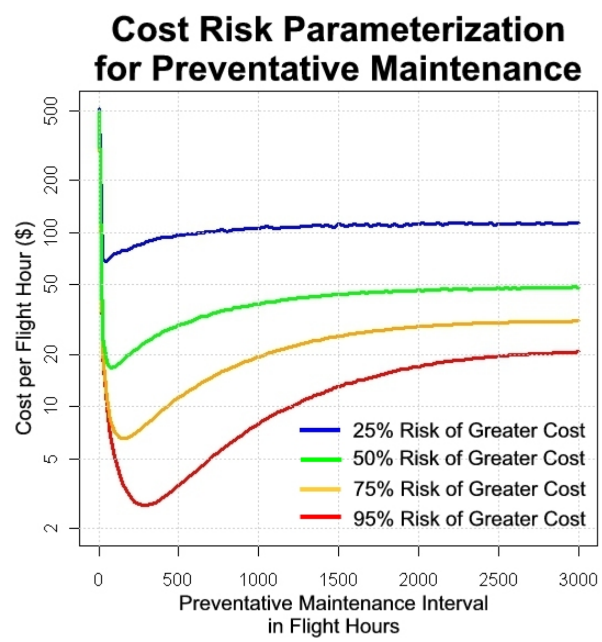

Figure 5 — Preventative maintenance costs appear to be minimal at all risk levels for preventative maintenance intervals between 50 and 350 flight hours.

Figure 5 offers some rather interesting insights. All of the cost risk lines have mostly leveled out by t_{pm} =2,500 flight hours. This suggests that there should not be much cost savings in using preventative maintenance with maintenance intervals beyond t_{pm} =2,500 flight hours. This is further confirmed by the 50% risk line converging to $49 per flight hours for preventative maintenance intervals t_{pm} >2,500 flight hours. This is the same cost at the 50% risk level for the current procedure of allowing the turbine to fail as shown in Figure 4. Also very interesting is that between about t_{pm} =50 flight hours and about t_{pm} =500 flight hours, all of the risk parameterizations seem to have minima. This suggests that selection of preventative maintenance intervals in this range may provide maximum cost savings at some reasonable level of risk.

Cost Savings Parameterized by Maintenance Interval

Using the samples of η and β given the data in Figure 2, samples of CS_{tpm}, the cost savings per flight hour provided by preventative maintenance for the C130 1500 Series Flight Deck Cooling Turbine can be parameterized as a function of preventative maintenance interval t_{pm} by evaluating equation (16) and using the procedure discussed in the section of this report on numerical methods. Figure 6 shows the cost savings per flight hour based on the data parameterized as a function of t_{pm} at two risk levels (red line – 5% risk that cost savings per flight hour will be less than the ordinate, blue line – 5% risk that cost savings per flight hour will exceed the ordinate) and the most likely cost savings per flight hour based on the data (the black line).

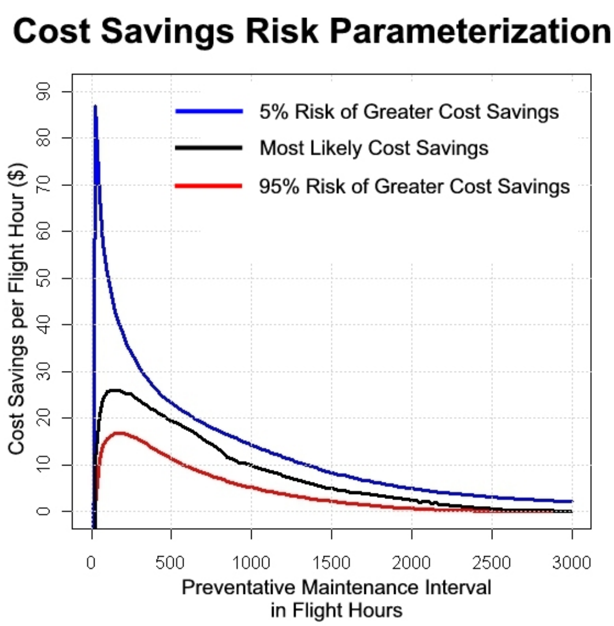

Figure 6 — The risks for cost savings per flight hour using a preventative maintenance interval show peaks at all risk levels for preventative maintenance intervals between 50 and 350 flight hours, the same region for the minima in Figure 5.

One way to interpret the red and blue lines in Figure 6 is that there is no more than a 10% risk that the cost savings per flight hour based on the data will be outside them; i.e., there is a 90% probability based on the data that the cost savings per flight hour will be between these lines. As an example, if a t_{pm} =500 flight hours is selected, then based on the data, there is a 90% probability that the cost savings per flight hour will be between \$12 and \$24. The most likely cost savings per flight hour for a t_{pm} =500 flight hours is just under \$20. The range in this 90% risk interval for preventative maintenance invervals t_{pm} >200 is actually quite tight. For preventative maintenance invervals t_{pm} < 200, the increase in span is to the high side of the most likely cost savings.

Figure 6 further confirms the insights noticed in Figure 5; beyond t_{pm} >2,500 flight hours, there appears to be, based on the data, very little chance of any cost savings, and the maximum cost savings at all risk levels appear to be between about t_{pm} =50 flight hours and about t_{pm} =500 flight hours. At a t_{pm} =250 flight hours, there is a 95% probability that the cost savings per flight hour based on the data will be greater than \$17, with the most likely cost savings of \$25. Figure 6 provides the decision maker with the risk information on cost savings based solely on the data that will enable a comfortable decision on selection of the preventative maintenance interval.

The annual cost savings for maintenance of the 1500 Series Flight Deck Cooling Turbine may be significant for the US Coast Guard, considering that each C130 is flown about 800 hours per year. With a hypothetical selection of a t_{pm} =250 flight hours, if only five C130 aircraft are equipped with the 1500 Series Flight Deck Cooling Turbine, there is no more than a 5% risk based on the data that annual maintenance cost savings will be less than \$68,000, and there is a 90% probability based on the data that annual maintenance cost savings will range between \$68,000 and \$144,000. If the entire USCG fleet of C130 aircraft is equipped with the 1500 Series Flight Deck Cooling Turbine, there is no more than a 5% risk based on the data that cost savings will be less than \$1.77M, and a 90% probability based on the data that annual maintenance cost savings will range between \$1.77M and \$3.744M. In this case, use of the balance of survivor data for the entire fleet should increase the annual cost savings at the same levels of risk.

4. CONCLUSIONS

Three important conclusions result from the work presented in this report.

First, a procedure has been developed for selecting an optimum cost preventative maintenance interval for high reliability aerospace systems. This procedure employs maximum objectivity without using any questionable assumptions. It also enables use of all available information including survivor data. When the replacement costs for allowing a unit to fail in service and costs for maintenance service are known, analytical and numerical methods provide distributions of cost savings risk per flight hour based on the data without use of any questionable assumptions. These cost savings risks can be parameterized as a function of candidate preventative maintenance intervals, enabling a decision maker to select an optimum cost preventative maintenance interval at a level of risk acceptable to them.

Second, even with very few actual failure data, the procedure developed in this report produces rather tight distributions of cost savings risk. For aerospace systems and subsystems designed to stringent reliability requirements, there may be few if any failure data and survivor data may outnumber failure data. The tight distributions of cost savings risk are perhaps non-intuitive, yet very welcome, considering results obtainable using Classical statistical methods on small event data sets.

Third, the procedure presented in this report may be applied directly to any aerospace system or subsystem with fixed costs for replacement and preventative overhaul, and may result in dramatic cost savings. Because of the inherent objectivity in the procedure, the lack of questionable assumptions, and the relatively small span of the 90% cost savings interval, decision makers should be much more comfortable selecting a preventative maintenance interval and enjoy cost savings immediately, instead of waiting for more failure data.

5. Acknowledgements

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BIOGRAPHY

Mark Powell has practiced Systems Engineering for over 35 years in a wide range of technical environments including DoD, NASA, DOE, and commercial. More than 25 of those years have been in the aerospace arena. His roles in these environments have included project manager, engineering manager, chief systems engineer, and research scientist. His academic affiliations have included the University of Idaho, Stevens Institute of Technology, and the University of Houston, Clear Lake. Mr. Powell maintains an active engineering and management consulting practice throughout North America, Europe, and Asia. Beyond consulting, he is sought frequently as a symposium and conference speaker and for training, workshops, and tutorials on various topics in Systems Engineering, Project Management, and Risk Management. Mr. Powell is an active member of AIAA, Sigma Xi, the International Society for Bayesian Analysis, and the International Council on Systems Engineering, where he serves as Assistant Director for Systems Engineering Processes.