Method to Employ Covariate Data in Risk Assessments

Mark A. Powell
Attwater Consulting
P.O. Box 57702
Webster, TX 77598-7702
208-521-2941
attwater@aol.com

Abstract—The International Space Station Program experienced a problem with the extra vehicular activity (EVA) pre-breathe oxygen (O_2) sensor on-board the International Space Station (ISS). [1] [2] Astronauts on the ISS must pre-breathe a mixture rich in oxygen prior to a scheduled EVA to prevent nitrogen narcosis (the bends) during the EVA. The O_2 measurement from this sensor on the pre-breathing apparatus was observed during its 270 day mission to be in error by more than ±6mmHg, subjecting the life of the EVA astronaut to an unacceptable risk. The ISS Program was faced with either halting EVA's for the ISS until this sensor could be redesigned, tested, and delivered to the ISS, or finding some other way to reduce this risk to the EVA astronauts to an acceptable level.

The method considered to reduce this risk was to compute average drift rates of the error of several O_2 sensors and to compensate the on-board measurements using these. Compensation of the experimental O_2 measurements using these average drift rates however produced errors in excess of ±6mmHg at less than 270 days since calibration, and even earlier than without drift compensation. This compensation scheme appeared to produce no risk reduction for the EVA astronaut, and possibly increased the risk. Quantification of the risk that O_2 sensor measurement errors would exceed the tolerable ±6mmHg range before the end of the 270 day mission period, with and without drift compensation, was required to enable a decision.

Time since calibration (TSC) is covariate with the O_2 sensor measurement error data. A proper distribution model that adds parameters can be used to take advantage of the covariate data for the O_2 sensor measurement errors. Unfortunately, no classical statistical methods exist to assess point estimates of risk using such a covariate distribution model. Conditional inferential methods, however, are suitable for quantified risk assessment using complex covariate models, and without using any assumptions. A conditional inferential process was developed to quantify the risks of these O_2 sensor measurement errors exceeding the ±6mmHg range before the end of the 270 day mission period, with and without drift compensation. The risk assessments thus obtained revealed that the risk of the O_2 sensor measurement error exceeding ±6mmHg was found to actually reduce by more than an order of magnitude using the proposed compensation scheme, with 95% assurance that the compensated risk would not exceed 1.5%. The ISS Program decided to employ the proposed drift compensation approach on-board the ISS.

For any aerospace risk assessment problem where measurement data have covariates, proper covariate models may be developed and conditional inferential methods may be employed to take advantage of the additional information provided by the covariates. These quantified risk assessments enable effective decision making.

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1. INTRODUCTION

Because space suits used on-board the Space Shuttle and International Space Station (ISS) use a much lower atmospheric pressure than that inside the vehicles, astronauts who are to perform extra vehicular activities (EVA) must pre-breathe an oxygen (O_2) rich mixture to avoid nitrogen narcosis, or the bends. Well known to divers, the bends can cause debilitating pain limiting the ability to function, and possibly cause irreparable injury or even death. Outside of the Space Shuttle or ISS, an astronaut performing an EVA who experiences the bends will be seriously compromised in performance of the EVA mission, and may be subjected to significant risk of injury or loss of life.

On board the ISS, the pre-breathe apparatus is equipped with an O_2 sensor to assure that the EVA astronaut is breathing the proper ratio of oxygen and nitrogen. If the O_2 sensor were to read more than 6mmHg above truth during the pre-breathe, the EVA astronaut is considered to be at an unacceptable risk of getting the bends during a normal EVA. If the O_2 sensor were to read more than 6mmHg

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below truth during the pre-breathe, the EVA astronaut would be breathing a mixture considered too oxygen rich and would subject the EVA astronaut to an unacceptable risk of oxygen toxicity. Oxygen toxicity can result in cell damage and death, with effects most often seen in the central nervous system, lungs and eyes. NASA personnel require that this O_2 sensor be accurate during its intended mission life to within ±6mmHg to avoid exposing EVA astronauts to unacceptable risks.

The author was alerted by NASA personnel that the oxygen sensor on the pre-breathe apparatus on the ISS had been observed to drift post calibration. This O_2 sensor is typically calibrated 90 days prior to launch and delivery to the ISS, and is expected to provide accurate service for 180 days on-board. The amount of drift in accuracy within this 270 day period since calibration that had been observed was greater than the maximum ±6mmHg required to avoid exposing EVA astronauts to unacceptable risks. In an analysis of the drift of several of these O_2 sensors, it was observed by NASA engineers that they all drifted in the same direction and about the same apparent rate. This raised the possibility of compensating the O_2 sensors for this drift rate, rather than requiring redesign, and postponement of EVA's on the ISS until the new design could be tested and delivered to the ISS.

NASA engineers performed a linear regression analysis on the drift error data obtained from five O_2 sensors to obtain the drift rate and intercept constants for the drift compensation scheme. The drift error data were then compensated using these constants. While the overall balance of errors looked better, some sensors showed errors of more than ±6mmHg even earlier than in the uncompensated data. It was not clear whether the drift compensation reduced the risk, or increased it.

For sensors that drift with time, the time since calibration (TSC) is a factor in the measurement errors that will be observed. TSC is thus covariate with the sensor measurement errors. A proper distribution model for the measurement errors will factor this covariate into the parameters of some standard distribution model, increasing the number of parameters in the resulting model. Unfortunately, no classical statistical methods exist that will allow point estimate assessment of risk using such covariate models. Fortunately, conditional inferential methods are particularly well suited to statistical inference for models that have covariate factors in the parameters.

This report describes the development of a covariate model for the ISS O_2 sensor data, and a conditional inferential approach to process the sensor measurement error data with covariate TSC to quantify the risk of the sensor measurement errors exceeding ±6mmHg both before and after drift compensation. The comparison of these risks before and after drift compensation allows determination of whether the drift compensation scheme was suitable as an alternative to sensor redesign and replacement.

2. METHOD

Model Selection

Selection of a model for the uncertainty about the data is a task typically based on knowledge of the physics that produces the data and the basic characteristics of the data. For this problem however, since linear regression was used to develop the drift compensation parameter values, it is appropriate to continue with the same assumptions necessary to apply linear regression. These assumptions are that the O_2 sensor error data are all independent and modelled by the same Normal distribution with parameters μ_s and σ_s.

To factor into this standard model the effects of the covariate TSC, the mean of the Normal distribution is modelled as a straight line as a function of TSC. Equation (1) provides this model for the mean of the Normal distribution.

$$\mu_s(\text{TSC}) = \mu_0 + \mu' * \text{TSC} \tag{1}$$

Equation (1) introduces two new parameters into the Normal model, μ_0 and μ', which results in a covariate Normal model. The covariate Normal model reflects the information provided by TSC for the sensor errors, and is presented in equation (2).

$$pd\left(e_s \mid \mu_0, \mu', \sigma_s, \text{TSC}\right) = \frac{1}{\sqrt{2\pi}\sigma_s} e^{-\left(\frac{1}{2}\right)\left(\frac{e_s - \mu_0 - \mu' * \text{TSC}}{\sigma_s}\right)^2} \tag{2}$$

Conditional Inferential Approach

The first step to assess the risk of exceeding the safe limits of ±6mmHg at 270 days since calibration is to infer from the data the uncertainty model for the parameters of the selected failure model, our covariate Normal distribution model in equation (2). This is where classical methods become inadequate; no classical statistical recipes have been developed for the covariate Normal model in equation (2). With conditional inferential methods, the joint probability density model for the parameters of the covariate Normal distribution model is developed based solely on the data. Where classical statistical recipes produce point estimates of the parameter values, conditional inferential methods produce full joint distributions of the parameters. This is particularly useful for risk assessments. Typical risk distributions are skewed, and point estimates of risk calculated from point estimates of distribution model parameters may be spurious.

With this joint density developed using conditional inferential methods, it is possible to compute any probability that might be useful, including the risk of exceeding the safe limits of ±6mmHg at 270 days since calibration. To develop the joint density of μ_0, μ', and σ_s given the data, Bayes' Law [1] is employed per equation (3) .

$$pd\left(\mu_0, \mu', \sigma_s \mid data\right) \propto pd\left(data \mid \mu_0, \mu', \sigma_s\right) pd\left(\mu_0, \mu', \sigma_s\right) \tag{3}$$

In equation (3),the first term to the right of the proportion, $pd(data|\mu_0, \mu', \sigma_s)$, is the *likelihood*. When the data is limited to observed events, this is the same likelihood function used in calculating maximum likelihood estimates of the parameters. The second term to the right of the proportion, $pd(\mu_0, \mu', \sigma_s)$, is the joint *prior* density for μ_0, μ', and σ_s. The joint *prior* density is selected to model the knowledge or ignorance of μ_0, μ', and σ_s before obtaining the data. The proportionality in the equation is insignificant; the proportionality constant can always be calculated by integrating over all values of μ_0, μ', and σ_s. The term on the left, $pd(\mu_0, \mu', \sigma_s|data)$, the joint density of μ_0, μ', and σ_s given the data, is called the joint *posterior* density.

Selection of the *prior* model for some problems can pose some difficulty. Some decision makers feel that using a priori knowledge of the parameters somehow prejudices the results, casting the pall of a rigged decision subject to second-guessing. Beyond that, for many uncertainty models that might be selected for the data for various problems, the parameters have no useful physical meaning, and thus no reason exists to have any a priori knowledge of them. To address both of these difficulties, it is possible to use a *prior* density model that imparts no a priori knowledge of the parameters. This is called using a *noninformative* or *ignorance prior* [2]. Use of *ignorance priors* establishes a basis of maximum objectivity for the decision, and alleviates the difficulty of dealing with any second-guessing. The joint *prior* density model is generally structured such that the parameters are independent. Using the Normal model, because μ_s(TSC) and σ_s are location and scale parameters respectively, Jeffrey's *priors* [3] are very suitable as the *ignorance priors* for μ_0, μ', and σ_s and are presented in equations (4).

$$\begin{aligned} pd\left(\mu_0\right) &\propto 1 \\ pd\left(\mu'\right) &\propto 1 \\ pd\left(\sigma_s\right) &\propto \frac{1}{\sigma_s} \end{aligned} \tag{4}$$

Now, given as data e_s(TSC), the O_2 sensor errors at the covariate TSC, the *posterior* density model is formed in equation (5) using the covariate Normal distribution with TSC_i being the time since calibration of the i^{th} sensor error measurement e_{si}.

$$pd\left(\mu_0, \mu', \sigma_s \mid data\right) \propto \prod_{i=1}^{N_{e_s}} \frac{1}{\sigma_s} e^{-\left(\frac{1}{2}\right)\left(\frac{e_{si}-\mu_0-\mu' * \text{TSC}_i}{\sigma_s}\right)^2} * \left(\frac{1}{\sigma_s}\right) \tag{5}$$

In equation (5), the first term to the right of the proportion is the *likelihood* for the O_2 sensor error data, the second term is the Jeffrey's *priors* for μ_0, μ', and σ_s. The *priors* for μ_0 and μ' do not appear explicitly.

Risk Distribution Formulation

The inference of the uncertainty distribution from the data of the risk of exceeding the safe limits of ±6mmHg at 270 days since calibration is very important. In any decision regarding compensation or redesign of the O_2 sensor, the decision should be based upon that risk, and whether it exceeds some unacceptable level and is appreciably improved (reduced) by the drift compensation scheme. The uncertainty distribution for that risk, $pd((R(|e_s|>6\text{mmHg})|\text{TSC})|data)$, can be developed by starting with the standard Normal distribution model denoted commonly as $\Phi(\cdot|\mu,\sigma)$. Equation (6) provides the formulation of this risk using this model where e_{max} is the absolute maximum acceptable O_2 sensor error of 6mmHg.

$$R\left(\left|e_s\right| > e_{\max} \mid \mu_s, \sigma_s\right) = P\left(\left|e_s\right| > e_{\max} \mid \mu_s, \sigma_s\right) = 2 * \Phi\left(-e_{\max} \mid \mu_s, \sigma_s\right) \tag{6}$$

By substituting equation (1) into equation (6), the formulation for the risk taking advantage of the covariate TSC is developed in equation (7).

$$R\left(\left|e_s\right| > e_{\max} \mid \mu_0, \mu', \sigma_s, \text{TSC}\right) = 2 * \Phi\left(-e_{\max} \mid \mu_0 + \mu' * \text{TSC}, \sigma_s\right) \tag{7}$$

For a given set of O_2 sensor errors, with their concomitant covariate times since calibration TSC, equation (5) provides the joint uncertainty model for μ_0, μ', and σ_s given this data. The uncertainty model for the risk of exceeding the maximum O_2 sensor error of 6mmHg based on the data, $pd((R(|e_s|>e_{max})|\mu_0, \mu', \sigma_s, \text{TSC})|data)$, is obtained by multiplying equation (7) by the posterior in equation (5). The full algebraic expansion of that product is quite busy, and as will be seen later, is unnecessary. The shorter form of this product is in equation (8).

$$pd\left(R\left(\left|e_s\right|>e_{max}\mid\mu_0,\mu',\sigma_s,\text{TSC}\right)\mid data\right)$$
$$=R\left(\left|e_s\right|>e_{max}\mid\mu_0,\mu',\sigma_s,\text{TSC}\right) \quad (8)$$
$$*\,pd\left(\mu_0,\mu',\sigma_s\mid data\right)$$

This still does not yield the quantity of interest, the uncertainty distribution of the O_2 sensor error as a function of TSC given the data, $pd((R(|e_s|>e_{max})|\text{TSC})|data)$. However, this distribution is obtained by applying marginalization integrals for μ_0, μ', and σ_s to equation (8). Equation (9) provides this marginalization without full algebraic expansion, which again will be shown later to be unnecessary.

$$pd\left(R\left(\left|e_s\right|>e_{max}\mid\text{TSC}\right)\mid data\right)$$
$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}R\left(\left|e_s\right|>e_{max}\mid\mu_0,\mu',\sigma_s,\text{TSC}\right) \quad (9)$$
$$*\,pd\left(\mu_0,\mu',\sigma_s\mid data\right)d\mu_0\,d\mu'\,d\sigma_s$$

By integrating the density model in equation (9), it is possible to obtain quantitative values for the assurance that the the risk of exceeding the safe limits of ±6mmHg at 270 days since calibration given the data is below a required level.

Numerical Methods

The uncertainty distribution in equation (5) is not analytically integrable. As a result, neither is the integral in equation (9), nor the marginalization integrals in equation (9). The solution is to use numerical methods, namely Monte Carlo methods [4]. Monte Carlo methods are used widely for accurately approximating the evaluation of probability integrals. Quite often, risk problems such as the subject of this report are solvable only using Monte Carlo methods.

The central issue to evaluating equations (5) and (9) using Monte Carlo methods is to obtain a large number of samples of μ_0, μ', and σ_s from the joint *posterior* uncertainty model in equation (5). There exist no statistical software packages with built-in samplers for the joint *posterior* density function of equation (5) based on the covariate Normal model. The remedy is to use Markov Chain Monte Carlo (MCMC) methods to sample this joint *posterior*. MCMC methods allow full range sampling of arbitrary distributions of any dimension given the formulation of the joint density [5]. With sufficient MCMC sampling of the joint *posterior* in equation (5), it is possible to compute very accurate approximations for almost any measure or statistic of interest, including evaluation of the integrals in equation (9) .

Once the joint MCMC samples of μ_0, μ', and σ_s are obtained, samples from the uncertainty model in equation (9) may be obtained using a non-intuitive yet simple process. Monte Carlo samples of $R(|e_s|>6\text{mmHg})|\text{TSC})|data)$ may be obtained by substituting equation (1) into equation (6) and simply evaluating at these joint MCMC samples of μ_0, μ', and σ_s. This simple process obviates the full algebraic expansion of equations (8) and (9).

With Monte Carlo samples of $R(|e_s|>6\text{mmHg})|\text{TSC})|data$, it is very easy to calculate such quantities as the assurance that the risk of the O_2 sensor error at $\text{TSC} = 270$ days does not exceed 5% given the data. Using M samples of $R(|e_s|>6\text{mmHg})|\text{TSC})|data$ (developed using the M joint samples of μ_0, μ', and σ_s), equation (10) shows how easily this quantity may be calculated.

$$P\left(R\left(\left|e_s\right|>e_{max}\mid\text{TSC}=270,\mu_s,\sigma_s\right)<0.05\mid data\right)$$
$$=\frac{\sum_{i=1}^{M}\begin{bmatrix}1\mid 2*\Phi\left(-e_{max}\mid\mu_{0i}+\mu_i'*270,\sigma_{si}\right)<0.05\\ 0\mid 2*\Phi\left(-e_{max}\mid\mu_{0i}+\mu_i'*270,\sigma_{si}\right)\geq 0.05\end{bmatrix}}{M} \quad (10)$$

3. DATA

Five O_2 sensors (units 1014, 1026, 1031, 1037 and 1039) were selected and calibrated, and allowed to drift for a period of almost 300 days. Measurements of O_2 concentration level accuracies for each sensor were observed at varying O_2 concentration levels several times during this period. Figure 1 shows these measurements with regression lines for each sensor. Also indicated in figure 1 are red horizontal lines at $e_{max} = \pm 6$mmHg, which is the maximum O_2 sensor error that is considered safe. Drifts accumulate to exceed this maximum O_2 sensor error as early as 80 days since calibration.

It was observed visually in figure 1 that all of the regression lines for all of the O_2 sensor errors had slopes of the same sign, and were close to the same value. This observation led to the consideration of compensation for the drift as an alternative to O_2 sensor redesign. All of the O_2 sensor error observations from all sensors were processed as a single group of data using linear regression to compute the intercept and slope of the drift with time since calibration. Figure 2 presents the resultant O_2 sensor errors compensated for drift using these values.

As can be seen in figure 2, there remain several times at which compensated O_2 sensor errors exceed ±6mmHg, and that these excursions beyond e_{max} occurred even earlier now than in figure 1 without compensation. At this point, it is unclear whether compensation for drift would be a suitable alternative to O_2 sensor redesign.

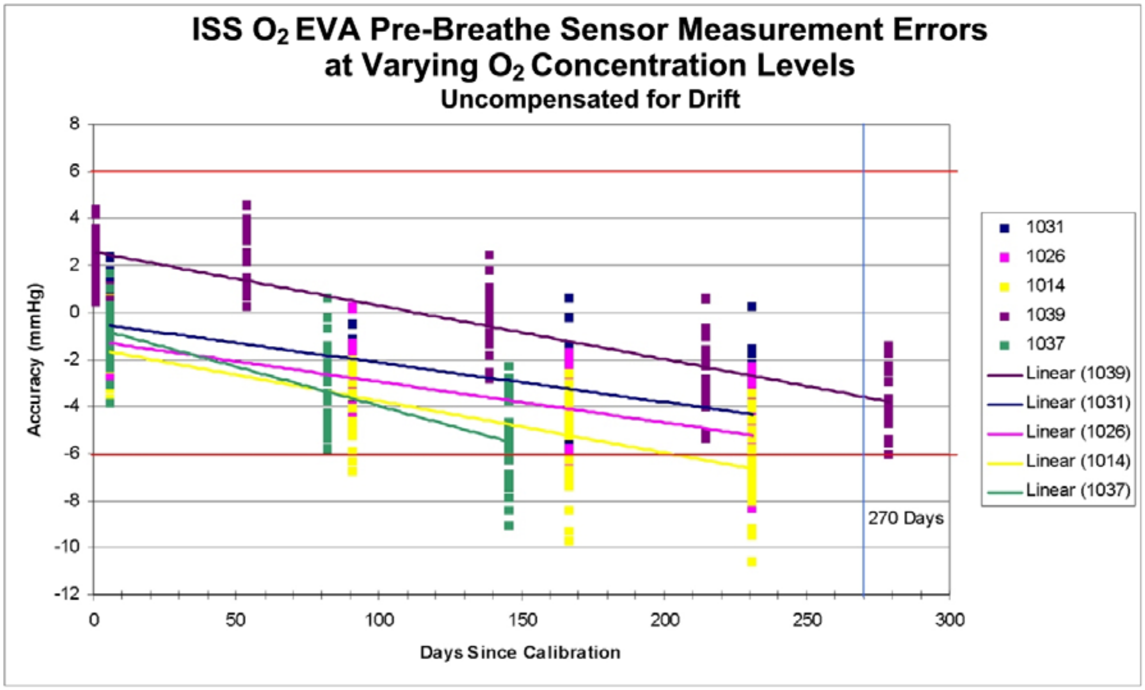

Figure 1 – Uncompensated ISS O_2 sensor measurement data demonstrate a consistent drift regardless of O_2 concentration level.

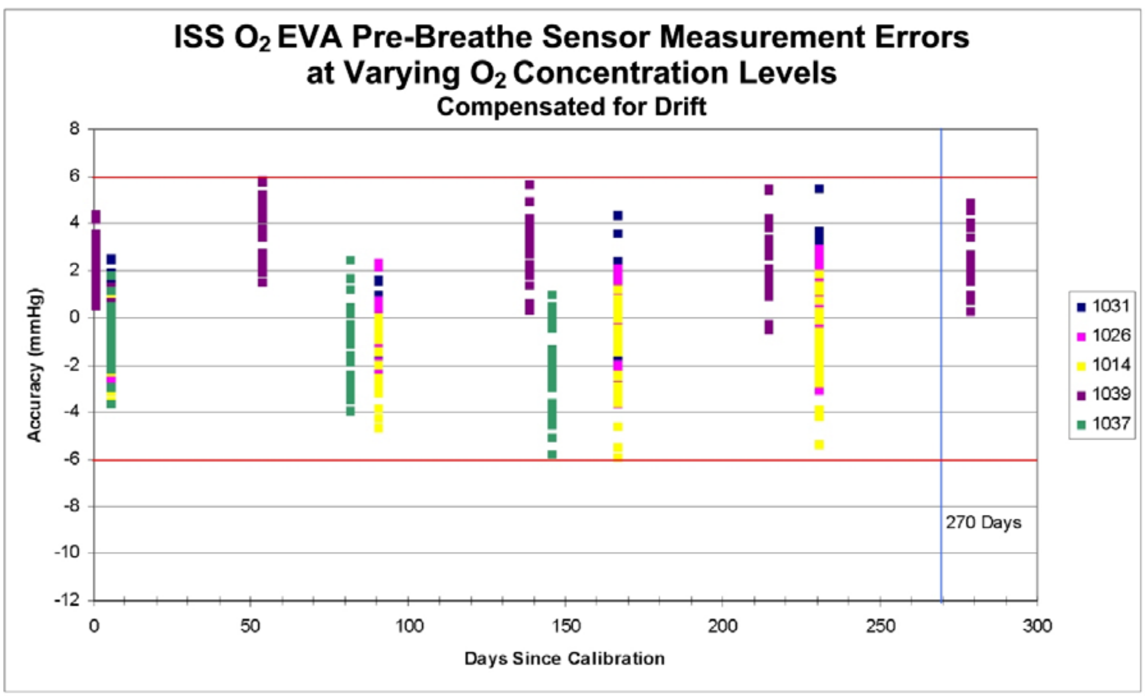

Figure 2 – ISS O_2 sensor measurement data compensated for drift still show that errors may exceed ±6mmHg, and perhaps even earlier than without compensation. Drift compensation may not have reduced the risk at all.

4. RESULTS

The first step in obtaining results is to use the procedure described in section 2 to obtain MCMC samples of μ_0, μ', and σ_s based on the uncompensated O_2 sensor error data in figure 1, and based on the compensated O_2 sensor error data in figure 2. The second step is to obtain the distributions of the risk of the O_2 sensor error exceeding ±6mmHg at 270 days since calibration given the data both for the uncompensated and compensated data.

MCMC Sampling of μ_0, μ', and σ_s

The procedure described in section 2 was applied to the data presented in figure 1 to obtain 10,000 MCMC samples of μ_0, μ', and σ_s without any compensation. Table 1 summarizes the statistics for the MCMC samples of μ_0, μ', and σ_s obtained using this procedure.

Table 1: MCMC Parameter Sample Statistics Obtained using a Conditional Inferential Approach on the Uncompensated O_2 Sensor Errors

	MCMC Sample Statistics			
Parameter	**Minimum**	**Maximum**	**Mean**	**σ**
μ_0	-1.07	0.14	-0.45	0.177
μ'	-0.023	-0.015	-0.019	0.001
σ_s	2.01	2.50	2.24	0.562

All of the Markov chains stabilized and were well behaved, and visual inspection of the marginal samples for each parameter revealed noise like behavior. No remarkable correlations were observed between any two parameters.

The procedure described in section 2 was applied to the data presented in figure 2 to obtain 10,000 MCMC samples of μ_0, μ', and σ_s with the proposed compensation. Table 2 summarizes the statistics for the MCMC samples of μ_0, μ', and σ_s obtained using this procedure for this data.

Table 2: MCMC Parameter Sample Statistics Obtained using a Conditional Inferential Approach on the Proposed Compensated O_2 Sensor Errors

	Sample Statistics			
Parameter	**Minimum**	**Maximum**	**Mean**	**σ**
μ_0	-1.05	0.32	-0.43	0.185
μ'	-0.00021	0.00756	0.004	0.001
σ_s	2.02	2.51	2.24	0.566

All of the Markov chains based on data with compensation stabilized and were well behaved, and visual inspection of the marginal samples for each parameter revealed noise like behavior. No remarkable correlations were observed between any two parameters.

Risk Calculations and Comparisons

The first issue related to this problem is assessment of the risk of the O_2 sensor errors exceeding ±6mmHg before 270 days without compensation. The drift compensation scheme only remains a viable solution if it reduces the risk. Recall from visual observations of figures 1 and 2 that it is difficult to tell how much or if any risk reduction is achieved by compensating the O_2 sensor for drift.

Monte Carlo samples of this risk were obtained by evaluating equation (7) at the joint MCMC samples of μ_0, μ', and σ_s obtained using the approach presented in section 2 for the uncompensated O_2 sensor error data in figure 1.

Likewise, the analog risk samples with the proposed drift compensation were obtained with the same procedure using the compensated O_2 sensor error data in figure 2.

A very good means to compare risk assessments is to use modified bar charts. Figures 3 and 4 use these modified bar charts. The bars in figures 3 and 4 start on the left at the 5^{th} quantile for the risk distribution, and end at the 95^{th} quantile. Based on the data for each bar, there is 90% certainty that the true value of the risk in question lies somewhere on the bar. The gray vertical lines on the bars are the modes of the distributions, and the color density on the bar is directly proportional to the probability density. These modified barcharts provide insights into these distributions visually. Figure 3 shows this comparison of risk distributions based on the data using a linear scale.

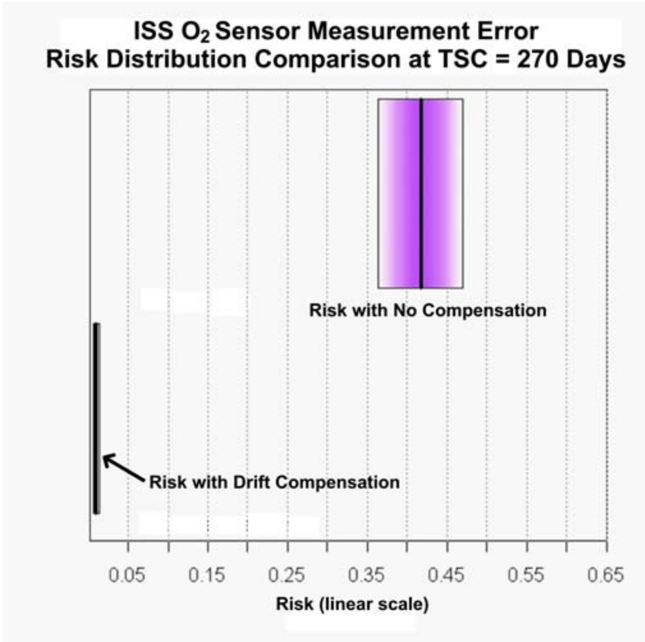

Figure 3 – There is a 90% probability based on the data that the risk of the uncompensated O_2 sensor error exceeding ±6mmHg by 270 days since calibration will be between 36% and 46%. The risk based on the data for the drift compensated O_2 sensor error is hardly measurable on this linear scale.

Figure 3 clearly demonstrates using the modified barcharts that the risk before drift compensation is rather serious at 270 days since calibration, and that drift compensation dramatically reduces it. This result was not anticipated from visual observation of figures 1 and 2. Since in figure 3 the bar for the risk distribution based on the data using drift compensation is so short, these same bars are presented with a logarithmic abscissa in figure 4 to show an expansion of the bar for risk with drift compensation.

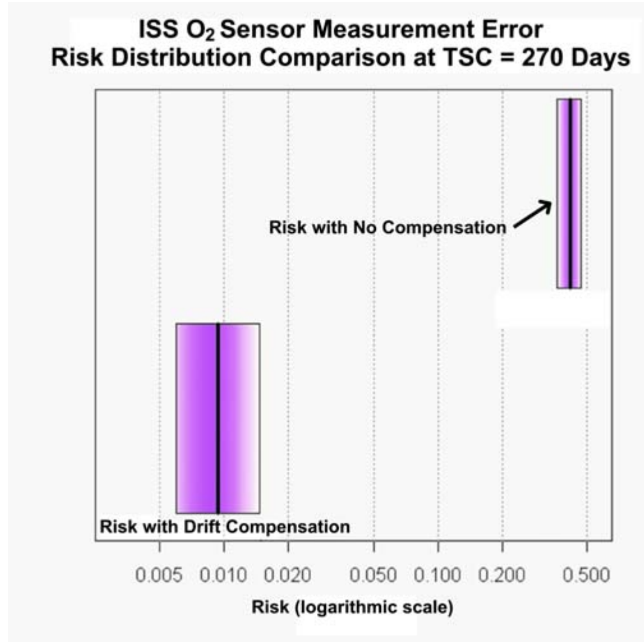

Figure 4 – There is a 90% probability based on the data that the risk of the drift compensated O_2 sensor error exceeding ±6mmHg at 270 days since calibration is between 0.6% and 1.5%. The logarithmic scale demonstrates the risk distribution after drift compensation.

While figure 3 made risk reduction using the proposed drift compensation scheme obvious, figure 4 helps quantify the amount of reduction. As seen in figure 4, there is a 95% probability based on the data that drift compensation for the O_2 sensor reduces the risk of exceeding ±6mmHg at 270 days since calibration is less than 1.5%. There is a 95% probability based on the data that the risk of exceeding ±6mmHg at 270 days since calibration without drift compensation is greater than 36%.

Where risk reduction was not so apparent in figures 1 and 2, figures 3 and 4, developed using the covariate model and conditional inferential method presented in section 2 to incorporate the covariate data for the O_2 sensor error data, demonstrate very clearly how much risk reduction actually occurred between figures 1 and 2.

This information helped the ISS program decide to use the proposed drift compensation for the pre-breathe O_2 sensor vice halting ISS EVA's until the pre-breathe O_2 sensor could be redesigned, tested, and delivered to the ISS.

5. CONCLUSIONS

There are two important conclusions from the work presented in this report.

First, it is no longer necessary to ignore relevant information from covariates in a risk assessment for aerospace system decisions. In making important decisions about aerospace systems, it is never a good idea to ignore available information. When using classical statistical recipes to obtain point estimates of risk, models cannot be used that take advantage of covariate data and the covariate data must be ignored. Rarely in fact are distribution models ever developed that formulate the parameters of a standard model as a function of the covariates because of this. The primary advance presented in this report is that conditional inferential methods can be used to develop risk inferences for such covariate models, as demonstrated for the ISS pre-breathe O_2 sensor measurement errors.

Second, decisions for aerospace systems are easier and more comfortable to make when the entire distribution of the risk is provided for the alternatives, rather than just point estimates. The modified barcharts that were used in figures 3 and 4 in this report demonstrate both a significant improvement in risk when drift compensation was used, as well as quantitative probabilistic values of the risk.

Every risk problem for aerospace systems is different. If, however, the data for a problem has covariates, then the parameters for an appropriate standard distribution model should be modifiable as functions of these covariates as demonstrated in this report. The choice of functions to incorporate the covariates should be based on engineering principles as well as understanding of the chosen standard model parameters. Further, these data with covariates may be processed using an approach based on conditional inferential methods that will produce full distributions of risk. Because the risk assessments took advantage of the information inherent in the covariates as well as the data, better decisions for aerospace systems are possible.

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BIOGRAPHY

Mark Powell has practiced Systems Engineering for over 35 years in a wide range of technical environments including DoD, NASA, DOE, and commercial. More than 25 of those years have been in the aerospace arena. His roles in these environments have included project manager, engineering manager, chief systems engineer, and research scientist. His academic affiliations have included the University of Idaho, Stevens Institute of Technology, and the University of Houston, Clear Lake. Mr. Powell maintains an active engineering and management consulting practice throughout North America, Europe, and Asia. Beyond consulting, he is sought frequently as a symposium and conference speaker and for training, workshops, and tutorials on various topics in Systems Engineering, Project Management, and Risk Management. Mr. Powell is an active member of AIAA, Sigma Xi, the International Society for Bayesian Analysis, and the International Council on Systems Engineering, where he has served as Risk Management Working Group Chair and as Assistant Director for Systems Engineering Processes.