Method for Investigating Repair/Refurbishment Effectiveness

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Abstract—Aerospace systems that fail in service are often repaired or refurbished and returned to service. [1] [2] [3] Repair/refurbishment may return the system to the equivalent of *new* condition, to some state *less than new* condition, or perhaps even to a *better than new* condition. Respectively, repair/refurbishment may have no effect on future reliability, degrade future reliability, or improve it. Depending on which reliability state the post-failure repair/refurbishment produces for the system, preventative maintenance schemes can differ dramatically. For example, should the repair/refurbishment return the system to a *less than new* condition with each subsequent failure, shorter preventative maintenance intervals as a function of number of the maintenance cycles increase overall availability and cost effectiveness. Should the repair/refurbishment return the system to a *better than new* condition, longer preventative maintenance intervals as a function of number of maintenance cycles increase overall availability and cost effectiveness.

The US Navy has provided a set of failure and survivor data for the F/A-18 E/F Super Hornet, General Electric F414 low bypass gas turbine engine removals due to foreign object damage. Such removals regularly repair/refurbish the engines and return them to service, often multiple times. No investigations into the reliability state post repair/refurbishment have been done, yet such knowledge could be factored into developing more effective preventative and corrective maintenance plans.

This report presents a method to determine whether repair/refurbishment cycles improve or reduce useful life post repair/refurbishment. This method employs a covariate Weibull model, where the scale and shape parameters are exponential functions of the covariate observed numbers of repair/refurbishment cycles. A conditional inferential approach is formulated using this model employing objective prior models to avoid any unnecessary assumptions. This model and method are then validated using simulated data to demonstrate that these phenomena may be reliably observed. The F/A-18 engine data are then processed to determine the state post repair/refurbishment, which may be used to enhance maintenance planning.

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1. INTRODUCTION

In earlier papers Millar, Mazzuchi and Sarkani [1] [2] report the results of a non-parametric statistical study of unscheduled engine removals data from records of operational service of the F/A-18 E/F Super Hornet, powered by a pair of General Electric F414 low bypass gas turbine engines. Engine removals are likely the most disruptive and costly maintenance action affecting naval aircraft, particularly if unscheduled on board an aircraft carrier in action at sea. The aircraft must be removed from flight operations to remove and replace the affected engine, the largest piece of equipment that can be swapped out this way. The affected engine is usually shipped to an intermediate maintenance base and a replacement spare engine is usually delivered to the ship. The engine is inspected and possibly tested at the intermediate maintenance base, and any modules requiring teardown for detail inspection and repair are replaced with spare modules. The modules to be serviced are forwarded to a maintenance depot to be rebuilt.

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Good understanding of the likelihood of unscheduled engine removals as a result of periodic inspection or damage detected in service is needed to optimize maintenance planning, facilities and manning, logistics and spares and parts provisioning. The purpose of this and the earlier studies is to characterize the pattern of engine removals for various causes as a function of accumulated engine operating hours. This information enables maintenance and logistics planning for unscheduled engine removals based on the history of the specific engines in service at a given location. For example, the probability of having to remove an engine from a specific aircraft due for inspection, for a specific reason for removal, can be better forecast with the methods developed during this study, based on the accumulated operating hours of the specific engines involved. Planning preventive and corrective maintenance over a longer horizon can be based on a firmer forecast of removals and the likely causes. Personnel, tools and equipment, logistics and spares can be provisioned in advance to speed turnaround and minimize costly spares stocks.

The earlier study yielded estimates for the hazard rate and survivor function through to overhaul for three classes of engine reasons for removal. The data used aggregated records from all F414 engines installed in F/A-18 aircraft over the first eight years of US Navy operational service. More detail on this study can be found in Millar [3].

One of the leading classes of reasons for removal was confirmed to be foreign object damage (FOD) to the engine fan or compressor, as detected through engine failure, pilot *squawk* or, most commonly, during periodic inspection at a more or less fixed interval. FOD alone caused about 20% of all unscheduled engine removals. The other two classes of reasons for removal combined multiple reasons for removal, had different statistical characteristics, and were considered to be qualitatively different in being more dependent on inherent engine component reliability and maintenance processes rather than exogenous influences.

FOD may result from bird ingestion in flight or, more commonly; it is due to debris sucked into the engines during operation on the ground (including shipboard) or during takeoff and landing. The exposure of individual aircraft to these flight conditions and thus FOD hazard is variable depending on operational location and mission. Over a large population of aircraft performing similar missions in a variety of environments, given the lack of detailed information on the exposure and usage on individual aircraft, our basic expectation was that aggregate FOD hazard levels should not vary greatly over the engine lifetime, i.e. FOD incidents and resultant removals might be expected to follow an exponential distribution. However, the results of the earlier studies of this data did not unambiguously confirm this expectation, motivating this study to further elucidate the findings and provide more useful guidance to F/A-18 propulsion system sustainment.

The data in this and previous studies included many engines that were removed, repaired, and returned to service. The number of removal/repair cycles associated with each datum is covariate with that datum, information that can be of significance in planning cost effective and mission effective preventative and corrective maintenance. The data in this and previous studies were also dominated by suspensions or survivors, engine removals for reasons such as for scheduled preventative maintenance when no damage was noted nor were repairs required. These survivor data had associated numbers of removal/repairs covariate data as well. To take advantage of the additional information provided by the number of repair/refurbishment cycle covariates associated with the failure and survivor data, the failure distribution model should reflect potential effects due to the covariate. In this report, a basic Weibull distribution model is modified to reflect changes in reliability as a function of the covariates. This covariate Weibull model expresses the scale and shape parameters as exponential functions of the covariate number of repair/refurbishment cycles, which allows each parameter to either increase or decrease as a function of repair/refurbishment cycles.

With such a covariate Weibull model, the next step is to infer from the data (the failures and survivors with their concomitant covariate numbers of repair/refurbishment cycles) the reliability as a function of repair/refurbishment cycle. An extensive literature search failed to reveal any classical method that could be used to infer reliability from covariate failure and survivor data using such a covariate distribution model. To address this problem, the authors developed a novel conditional inferential approach that could infer reliability from covariate failure and survivor data using the covariate Weibull model.

Before attempting to infer reliability for the F/A-18 engine using the covariate Weibull model and the conditional inferential approach developed by the authors, both the covariate Weibull model and conditional inferential approach were validated using simulated covariate failure and survivor data and the validation process and results are presented in this report. These simulated data reflected a complex relationship between reliability as a function of repair/refurbishment cycle, perhaps more complex than would be expected from a real world scenario. Obtaining successful validation results using this simulated data indicates a fairly robust validation of both the covariate Weibull model and conditional inferential approach.

The data from the F/A-18 engines in this report consisted of 238 failure data and 593 survivor data with associated repair/refurbishment cycle number covariate data. These data were processed using the covariate Weibull model and the conditional inferential approach developed for it, and some rather interesting and unexpected results were obtained in the inference for reliability as a function of repair/refurbishment cycle number. Not so unexpected was that as repair/refurbishment cycle number increased, the failure mode trended from *early wearout* to *infant mortality* (see section 2). A surprise inference was that the critical life increased as repair/refurbishment cycle number increased.

This report concludes with a hypothetical demonstration of how these inferences obtained using the covariate Weibull model via the conditional inferential method as described could be used in improving mission assurance via preventative maintenance scheduling taking advantage of these results.

2. METHOD

General Failure Model Selection

Selection of a model for the uncertainty about the data is a task based on knowledge of the physics that produces the data, and the basic characteristics of the data. The data to be analyzed for reliability related problems consists of times of failure, and times at which the unit was observed to have not failed as yet, commonly called survivors or suspensions. These data are one-sided; they can only have positive semi-definite values. Further, there is no reason to suspect that the physics involved in failure would produce a multi-modal time of failure model. Beyond these two facts, nothing more can be presumed about the distribution of failure times.

Equation (1) provides the general Weibull density function, which has a location parameter $t_1 \geq 0$, a scale parameter $\eta > 0$, and a shape parameter $\beta > 0$.

$$pd(t_f \mid t_1, \eta, \beta) = \left(\frac{\beta}{\eta}\right)\left(\frac{t_f - t_1}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_f - t_1}{\eta}\right)^{\beta}} \quad (1)$$

The Weibull model is a very general model for reliability related problems. One very useful feature of the Weibull model for reliability problems is that the parameters all have physical meanings. This is not the case for many probability distribution models. The location parameter t_1 represents the time before which failures cannot occur, and is called the *failure-free time*. The scale parameter η is the time at which 63.2% of all failures will have occurred, and is called the *critical life*. The shape parameter β is an indicator of failure mode. Values of $\beta < 1$ indicate an *infant mortality* failure mode. Values of $\beta = 1$ indicate a *useful life* failure mode. Values $1 < \beta < 4$ indicate an *early wearout* failure mode. And, values of $\beta > 4$ indicate an *old age* failure mode. Depending on the values of these parameters, the Weibull model can represent just about any uni-modal, one-sided distribution shape for failures imaginable, with skews to either left or right.

An important aside relative to this density formulation: Weibull's original paper [4] published in September 1951 provided a distribution function that would produce the density function in equation (2).

$$pd(t_f \mid t_1, \lambda, \alpha) = \left(\frac{\alpha}{\lambda}\right)\left(t_f - t_1\right)^{\alpha-1} e^{-\frac{\left(t_f - t_1\right)^{\alpha}}{\lambda}} \quad (2)$$

In discussions of Weibull's paper [5] published in June 1952, Weibull noted that his distribution function as originally published was incorrect by stating that the "…parentheses are an awkward misprint." Correction of this misprint produces the density function in equation (1).

The significance of this typographical error is profound. First, equation (2) cannot be reparameterized to produce equation (1) without comingling the parameters; the density function in equation (2) is fundamentally flawed since neither λ nor α can be classed as proper location, scale, or shape parameters. Second, textbooks [6] [7] exist that use the incorrect density function in equation (2) for the Weibull model. And, third, there are statistical software packages and tools [8] [9] that use the incorrect density function in equation (2) for the Weibull model. The caveat for the reader of this report is that whenever encountering any work using the Weibull model, and when considering any software package or tool, it is imperative to verify that the implementation of the Weibull model uses the proper form expressible as equation (1). The results obtained in any analytical work or through use of a software package that uses a form expressible as equation (2) subsequently may be pathological.

For the work presented in this report, the location parameter t_1 in equation (1) is set to zero. There exists no reason to believe that any engine could not fail the instant operation begins.

Incorporating a Repair/Refurbishment Number Covariate

If a repair/refurbishment is known to return a system to a new condition, then failure or survivor data obtained post repair/refurbishment should be modelled exactly as for new systems. Should there be uncertainty as to whether a repair/refurbishment will return the system to a *new* condition or some other condition, then the number of repair/refurbishments for any particular failure or survivor datum is covariate for that datum. The failure model should thus reflect the effect of the covariate upon the parameters of the model.

For the Weibull model, this means that the covariate should be implemented such that it can change the base values for the parameters η and β as the covariate value changes, yet not reduce either parameter to an illegal value of zero or less. A suitable mathematical implementation to achieve this is to formulate η and β as exponential functions of the number of repair/refurbishments, N_r, introducing four new parameters, η_0, β_0, η_c, and β_c. Equations (3) provide such suitable formulations.

$$\eta(N_r) = \eta_0 * e^{\eta_c * N_r}$$
$$\beta(N_r) = \beta_0 * e^{\beta_c * N_r} \tag{3}$$

Substitution of $\eta(N_r)$ and $\beta(N_r)$ from equations (3) for η and β respectively into equation (1) produces a failure model that can reflect the effects of repair/refurbishments. Note also in equations (3) that if the new parameters $\eta_c = \beta_c = 0$, then this covariate model reduces to the original Weibull model in equation (1). Unlike η_0 and β_0, which have the same limits as η and β, η_c and β_c can take any value $(-\infty,\infty)$.

Conditional Inferential Approach

The first step to investigate whether the numbers of repair/refurbishments affect the reliability of an aerospace system is to infer from the data the uncertainty model for the parameters of the selected failure model, our covariate Weibull distribution model. With conditional inferential methods, the joint probability density model for the parameters of the covariate Weibull distribution is developed based solely on the data. With this joint density, it is possible to compute any probability calculation that might be useful. To develop the joint density of η_0, β_0, η_c, and β_c given the data, Bayes' Law [10] is employed per equation (4).

$$pd(\eta_0, \beta_0, \eta_c, \beta_c \mid data) \propto pd(data \mid \eta_0, \beta_0, \eta_c, \beta_c)\, pd(\eta_0, \beta_0, \eta_c, \beta_c) \tag{4}$$

In equation (4), the first term to the right of the proportion, $pd(data|\eta_0, \beta_0, \eta_c, \beta_c)$, is the *likelihood*. When the data is limited to only failure times, this is the same likelihood function used in calculating maximum likelihood estimates. The second term to the right of the proportion, $pd(\eta_0, \beta_0, \eta_c, \beta_c)$, is the joint *prior* density for η_0, β_0, η_c, and β_c. The joint *prior* density is selected to model the knowledge or ignorance of η_0, β_0, η_c, and β_c before obtaining the data. The proportionality in the equation is insignificant; the proportionality constant can always be calculated by integrating over all values of η_0, β_0, η_c, and β_c. The term on the left, $pd(\eta_0, \beta_0, \eta_c, \beta_c|data)$, the joint density of η_0, β_0, η_c, and β_c. given the data, is called the joint *posterior* density.

Selection of the *prior* model for some problems can pose some difficulty. Some decision makers feel that using a priori knowledge of the parameters somehow prejudices the results, casting the pall of a rigged decision subject to second-guessing. Beyond that, for many uncertainty models that might be selected for the data for various problems, the parameters have no useful physical meaning, and thus no reason exists to have any a priori knowledge of them. To address both of these difficulties, it is possible to use a *prior* density model that imparts no a priori knowledge of the parameters. This is called using a *noninformative* or *ignorance prior* [11]. Use of *ignorance priors* establishes a basis of maximum objectivity for the decision, and alleviates the difficulty of dealing with any second-guessing. The joint *prior* density model is generally structured such that the parameters are independent. Using the Weibull model, because $\eta(N_r)$ and $\beta(N_r)$ are scale and shape parameters respectively, Jeffrey's *priors* [12] are very suitable as the *ignorance priors* for η_0 and β_0 and are presented in equations (5).

$$pd(\eta_0) \propto \frac{1}{\eta_0}; \;\; pd(\beta_0) \propto \frac{1}{\beta_0} \tag{5}$$

On the other hand, η_c, and β_c are arbitrary constants in our covariate Weibull distribution model. A constant density uncertainty model reflects objectivity for such parameters, and the *ignorance priors* for η_c, and β_c are given in equation (6).

$$pd(\eta_c) \propto 1 \;\; ; \;\; pd(\beta_c) \propto 1 \tag{6}$$

Now, given as data N_f failures and N_s survivors (times of good inspections or when some other unrelated failure occurred – e.g., a maintenance mechanic breaks off a stud in inspection), the *posterior* density model is formed in equation (7) using the covariate Weibull distribution from equation (4) with TSR_{fi} being the time since repair/refurbishment of the i^{th} failure datum with covariate number of repairs/refurbishments N_{rfi}, and TSR_{sj} being the time since repair/refurbishment of the j^{th} survivor datum with covariate number of repairs/refurbishments N_{rsj}.

$$pd(\eta_0, \beta_0, \eta_c, \beta_c \mid data) \propto \left[\prod_{i=1}^{N_f} \left(\frac{\beta_0 * e^{\beta_c * N_{rfi}}}{\eta_0 * e^{\eta_c * N_{rfi}}} \right) \left(\frac{TSR_{fi}}{\eta_0 * e^{\eta_c * N_{rfi}}} \right)^{\left(\beta_0 * e^{\beta_c * N_{rfi}} - 1\right)} * e^{-\left(\frac{TSR_{fi}}{\eta_0 * e^{\eta_c * N_{rfi}}} \right)^{\beta_0 * e^{\beta_c * N_{rfi}}}} \right] * \left[\prod_{j=1}^{N_s} e^{-\left(\frac{TSR_{sj}}{\eta_0 * e^{\eta_c * N_{rsj}}} \right)^{\beta_0 * e^{\beta_c * N_{rsj}}}} \right] * \left(\frac{1}{\eta_0} \right) * \left(\frac{1}{\beta_0} \right) \tag{7}$$

In equation (7), the first term to the right of the proportion in brackets ([·]) is the *likelihood* for the covariate failure data, the second term in brackets ([·]) is the *likelihood* for the covariate survivor data, and the two remaining terms are the Jeffrey's *priors* for η_0 and β_0. The *priors* for η_c, and β_c do not appear explicitly. One very nice feature of conditional inferential methods apparent in equation (7) is that survivor data can be used directly via the likelihood [13]. Observed times at which a subsystem in service has not failed comprise very important information that should not be neglected in the posterior or in the decision. For some problems, the number of survivor data may exceed that for failure data, and there may be only survivor data and no failure data at all. Conditional inferential methods provide solutions for these data sets [14]; solutions are not possible using classical methods without employing numerous assumptions that may be questionable and adding appreciable conservatism [14].

Reliability Distribution Formulation

The inference from the data of the uncertainty distribution for the reliability as a function of the number of repair/refurbishments, N_r, can play a significant role in developing a maintenance schedule that maximizes mission assurance or minimizes cost of maintenance. The uncertainty distribution $pd(R(T\,|N_r)|data)$ can be developed by starting with the standard Weibull model. Equation (8) provides the formulation of reliability using this model.

$$R(T \mid \eta, \beta) = e^{-\left(\frac{T}{\eta}\right)^{\beta}} \tag{8}$$

For the covariate Weibull distribution model, reliability becomes a function of N_r by a simple substitution of equations (3) into equation (8), as is displayed in equation (9).

$$R(T \mid N_r, \eta_0, \beta_0, \eta_c, \beta_c) = e^{-\left(\frac{T}{\eta_0 * e^{\eta_c * N_r}}\right)^{\beta_0 * e^{\beta_c * N_r}}} \tag{9}$$

For a given set of failure and survivor data, with their concomitant covariate numbers of repairs/refurbishments, equation (7) provides the joint uncertainty model for η_0, β_0, η_c, and β_c given this data. The uncertainty model $pd(R(T|N_r,\eta_0,\beta_0,\eta_c,\beta_c)|data)$ is obtained by multiplying equation (9) by the posterior in equation (7). The full algebraic expansion of that product is quite busy, and as will be seen later, is unnecessary. The shorter form of this product is in equation (10).

$$\begin{aligned} pd\left(R(T \mid N_r, \eta_0, \beta_0, \eta_c, \beta_c) \mid data\right) \\ = R(T \mid N_r, \eta_0, \beta_0, \eta_c, \beta_c) \\ * \, pd\left(\eta_0, \beta_0, \eta_c, \beta_c \mid data\right) \end{aligned} \tag{10}$$

This still does not yield the quantity of interest, the uncertainty distribution of reliability given the data as a function of repair/refurbishment number $pd(R(T|N_r)|data)$. However, this distribution is obtainable by applying marginalization integrals for η_0, β_0, η_c, and β_c. to equation (10). Equation (11) provides this marginalization.

$$\begin{aligned} & pd\left(R(T \mid N_r) \mid data\right) \\ & = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} pd\left(R(T \mid N_r, \eta_0, \beta_0, \eta_c, \beta_c) \mid data\right) \\ & \qquad * \, d\eta_0 \, d\beta_0 \, d\eta_c \, d\beta_c \\ & = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} R(T \mid N_r, \eta_0, \beta_0, \eta_c, \beta_c) \\ & \qquad * \, pd\left(\eta_0, \beta_0, \eta_c, \beta_c \mid data\right) d\eta_0 \, d\beta_0 \, d\eta_c \, d\beta_c \end{aligned} \tag{11}$$

Equation (11) also would be quite busy with full algebraic expansion, but such is unnecessary as will be seen later. By integrating the density model in equation (11), quantitative values of the assurance based on the data that the reliability exceeds some required level for the system given N_r. Another way to say this is that by integrating equation (11), one can calculate quantitatively how sure they can be based on the data that the reliability exceeds a required level given N_r. The compliment of this assurance is the risk that the reliability of the system, given the data for a specified number of repair/refurbishments, will not meet the required value.

Numerical Methods

The uncertainty distribution in equation (7) is not analytically integrable. As a result, neither is the integral in equation (10) analytically integrable, nor would equation (11) be. The solution is to use numerical methods, namely Monte Carlo methods [15]. Monte Carlo methods are used widely for accurately approximating the evaluation of probability integrals. Real world risk problems, such as the subject of this report, are quite often solvable *only* using Monte Carlo methods.

The central issue to integrating equations (7) and (11), using Monte Carlo methods, is to obtain a large number of samples of η_0, β_0, η_c, and β_c from the joint *posterior* uncertainty model in equation (7). There exist no statistical software packages with built-in samplers for the joint *posterior* density function of equation (7). The remedy is to use Markov Chain Monte Carlo (MCMC) methods to sample this *posterior*. MCMC methods allow full range sampling of arbitrary distributions of any dimension given the formulation of the joint density [16]. With sufficient MCMC sampling of the joint *posterior* in equation (7), it is possible to compute very accurate approximations for almost any measure or statistic of interest, including integration of the uncertainty distribution of reliability given the data as a function of repair/refurbishment number, $pd(R(T|N_r)|data)$, in equation (11).

Once the joint MCMC samples of η_0, β_0, η_c, and β_c are obtained, samples from the uncertainty model in equation (11) then may be obtained using a non-intuitive yet simple process. Monte Carlo samples of $R(T|N_r)|data$ are obtained by simply evaluating equation (9) at these joint MCMC samples of η_0, β_0, η_c, and β_c. at the desired value of N_r. It is this simple process that obviates full algrebraic expansion of equations (10) and (11).

Using the Monte Carlo samples of $R(T|N_r)|data$, it is very easy to calculate such quantities as the risk that the reliability at $T = 2$ does not exceed 90% after two repair/refurbishments given the data. Using M samples of $R(T|N_r)|data$ (developed using the M joint samples of η_0, β_0, η_c, and β_c), it is only necessary to count the number of samples of $R(T=2|N_r = 2)|data < 0.9$ and divide by M. Equation (12) shows how easily this risk may be calculated by evaluating equation (9) at the joint samples of η_0, β_0, η_c, and β_c for $N_r = 2$ and performing this counting process.

$$P\left(R\left(T=2\,|\,N_r=2\right)<0.9\,|\,data\right) = \frac{\sum_{i=1}^{M}\begin{bmatrix} 1 \,\Big|\, \left(e^{-\left(\frac{2}{\eta_{0i} * e^{\eta_{ci} * 2}}\right)^{\beta_{0i} * e^{\beta_{ci} * 2}}} < 0.9 \right) \\ 0 \,\Big|\, \left(e^{-\left(\frac{2}{\eta_{0i} * e^{\eta_{ci} * 2}}\right)^{\beta_{0i} * e^{\beta_{ci} * 2}}} \geq 0.9 \right) \end{bmatrix}}{M} \quad (12)$$

3. VALIDATION

Before using the covariate Weibull model and conditional inferential approach presented in section 2 on the F/A-18 engine covariate failure and survivor data, it is prudent to validate the approach by using artificial data generated using known values of η_0, β_0, η_c, and β_c for various values of N_r. 1,000 failure and survivor data were developed for each covariate value of $N_r = 0$, 1, 2, and 3 (total of 4,000 covariate data) by sampling the Weibull model that results when equations (3) are substituted into equation (1) using known values of η_0, β_0, η_c, and β_c. Table 1 contains the values of η_0, β_0, η_c, and β_c used for validation.

Table 1: Validation Model True Parameter Values

Parameter	Value
η_0	300
β_0	1.5
η_c	-0.1014
β_c	-0.2747

For this validation process, the values in Table 1 will cause the *critical life* to reduce as the number of repairs/refurbishments increases, and the failure mode will shift from *early wearout* to *infant mortality* as well. Figure 1 demonstrates how the true validation failure data distributions change as values of N_r change from 0 to 3.

Figure 1 – True Validation Failure Distributions for $N_r = 0$ (red), 1 (orange), 2 (green), and 3 (blue) reflect hypothetical decreases in *critical life* with failure modes moving from *early wearout* more to *infant mortality* as N_r increases.

The validation data were generated by the following process, assuming that preventative maintenance would occur at TSR = 400. With $N_r = 0$, 1,000 failure samples were generated from the Weibull distribution with parameter values for η and β developed from the parameter values for η_0, β_0, η_c, and β_c from Table 1 using equations (3). Samples that occurred before TSR = 400 were collected as failures with covariate $N_r = 0$, and those that occurred at or after TSR = 400 were collected as survivors at 400 with covariate $N_r = 0$. This step was repeated with covariate values ranging from $N_r = 1$ to $N_r = 3$. Table 2 shows the numbers of samples collected for each covariate value.

Table 2: Validation Failure and Survivor Sample Numbers by Covariate Value

N_r	0	1	2	3
Failures	794	798	773	792
Survivors	206	202	227	208

Figure 2 shows the empirical distributions of the samples of validation failures for each covariate value.

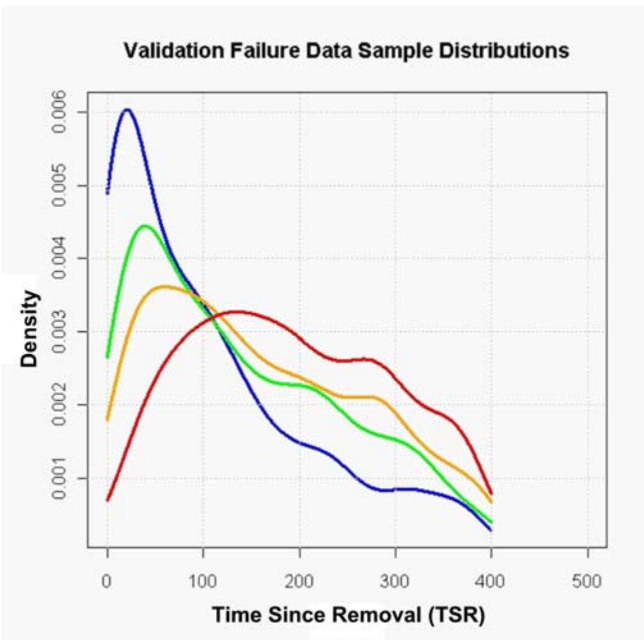

Figure 2 – Distributions of validation failure sample data for $N_r = 0$ (red), 1 (orange), 2 (green), and 3 (blue) demonstrate how the sampled failure distributions changed with successive repair/refurbishments.

In figure 2, the codes to produce these density curves uses a cosine bell function on the ends of the data ranges, which for $N_r = 3$ (green) and $N_r = 4$ (blue) created distortions of the true modes at TSR = 0 that caused them to return to zero instead of the true value of ∞. The fact that all failures beyond TSR = 400 were used as survivor data effectively truncates the failure data sampling possible from the distributions in figure 1. This truncation combined with the cosine bell function as applied by the codes produces the wavy artifacts out past TSR = 200. The distributions in figure 2 actually look more like real data distributions than those in figure 1, which should make these data suitable for a robust validation of the approach presented in section 2.

Table 3 provides the statistics for the MCMC samples of the parameters generated by using the procedure outlined in section 2 with the validation data as described in Table 2 and figure 2.

Table 3: Validation Model Parameter Sample Statistics with True Parameter Values in Parentheses below the Mean Values

	Validation Sample Statistics			
Parameter	**Minimum**	**Maximum**	**Mean (True)**	**σ**
η_0	273.9	318.6	295.4 (300)	6.58
β_0	1.38	1.66	1.49 (1.5)	0.039
η_c	-0.162	-0.040	-0.105 (-0.1014)	0.017
β_c	-0.315	-0.214	-0.264 (-0.2747)	0.014

In Table 3, the mean values of the samples of all of the parameters compared very favorably with the true values of the parameters listed in Table 1, and the standard deviations were all relatively small. The Markov chains were all very stable, and visual inspection of the joint parameter samples revealed that the sample ensembles appeared to be noise. These results validate that the approach presented in section 2 will allow investigation of reliability for systems with failure and survivor data with multiple repair/refurbishments covariates.

4. DATA

The failure and survivor data with their concomitant covariate number of repair/refurbishments collected for the F/A-18 engines are summarized in Table 4 and Figure 3.

Table 4: F/A-18 Engine Failure and Survivor Data Numbers for Covariate Values $N_r = 0, 1, 2, 3$, and 4.

N_r	0	1	2	3	4	Totals
Failures	193	41	3	1	0	238
Survivors	421	140	29	2	1	593

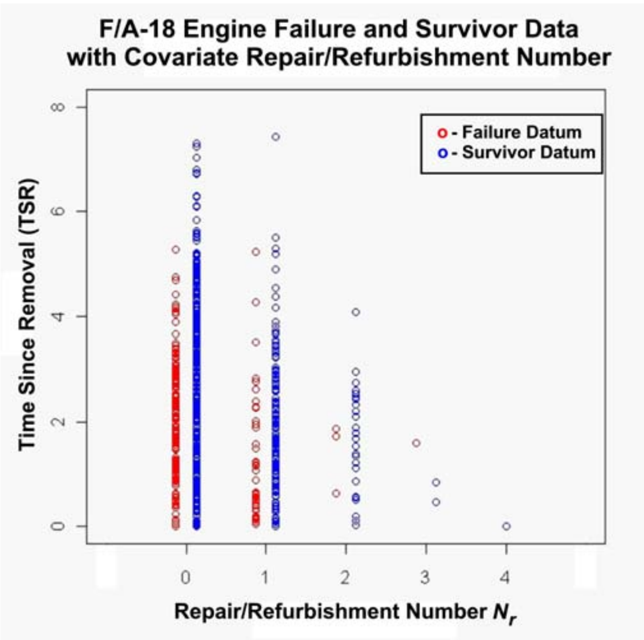

Figure 3 – F/A-18 engine failure (red) and survivor (blue) data for $N_r = 0, 2, 3$, and 4.

Per Table 4, there were a total of 238 failure data with covariates, and a total of 593 survivor data with covariates. The covariate numbers ranged up to $N_r = 4$; however, with $N_r = 4$, there was only one survivor and no failures. The numbers of data dropped off rather quickly as the covariate number increased. Figure 3 shows clearly that these data are dominated by survivor data, and the distribution of failures appeared to change dramatically between covariate values of $N_r = 0$ and $N_r = 1$. To protect proprietary corporate data, the times of failures and survivors in figure 3 were multiplied by an arbitrary constant yielding arbitrary *Time Since Removal* (TSR) units. These arbitrary TSR units are used throughout this report when discussing results obtained from the data.

5. RESULTS

The procedure described in section 2 was applied to the F/A-18 engine data described in section 4 to obtain 10,000 joint samples of η_0, β_0, η_c, and β_c. These joint samples of the covariate Weibull model will be discussed initially, with a few insights gained directly therefrom. Second, results such as reliability that can be derived from these joint samples will be discussed. Third, these results will be used in a demonstration of how a preventative maintenance schedule can be developed to improve mission assurance.

MCMC Sampling Based on the Data

The F/A-18 engine failure and survivor data with covariate data were processed with the procedure described in section 2, and 10,000 joint samples of η_0, β_0, η_c, and β_c were obtained. Table 5 provides the marginal parameter sample statistics that were obtained for η_0, β_0, η_c, and β_c.

Table 5: F/A-18 Engine Covariate Weibull Model Parameter Sample Statistics

	Parameter Sample Statistics			
Parameter	**Minimum**	**Maximum**	**Mean**	**σ**
η_0	4.77	7.47	6.06	0.407
β_0	1.09	1.56	1.33	0.079
η_c	-0.216	1.127	0.303	0.210
β_c	-0.693	0.211	-0.220	0.126

Figures 4-7 provide the marginal distributions of the 10,000 joint samples of η_0, β_0, η_c, and β_c respectively.

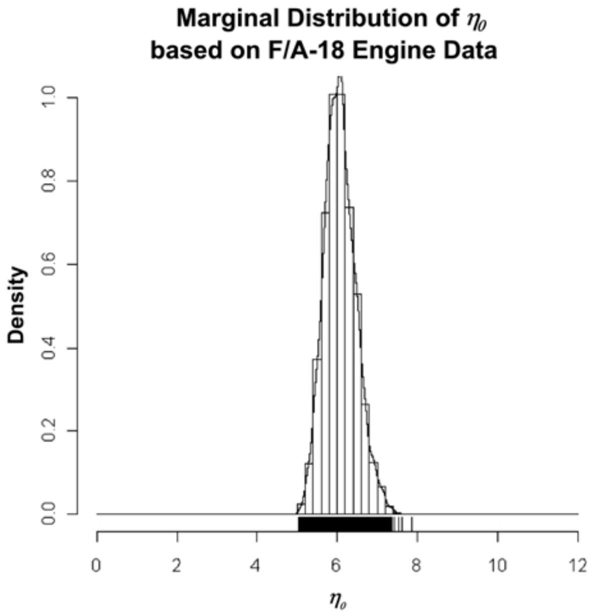

Figure 4 – The marginal distribution of the 10,000 MCMC samples of η_0 obtained from the F/A-18 engine data indicates a relatively tight clustering about $\eta_0 = 6$.

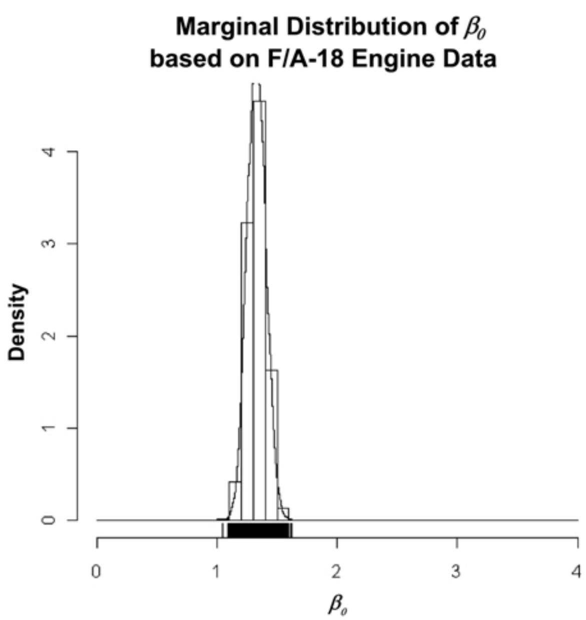

Figure 5 – The marginal distribution of the 10,000 MCMC samples of β_0 obtained from the F/A-18 engine data suggests that the engines have very little risk of having an *infant mortality* failure mode before the first repair/refurbishment.

Figures 4 and 5 show rather sharp marginal distributions based on the data for both base Weibull parameters η_0 and β_0. The base failure mode represented by the marginal distribution of β_0 indicates that the failure mode before any repair/refurbishments is *early wearout*.

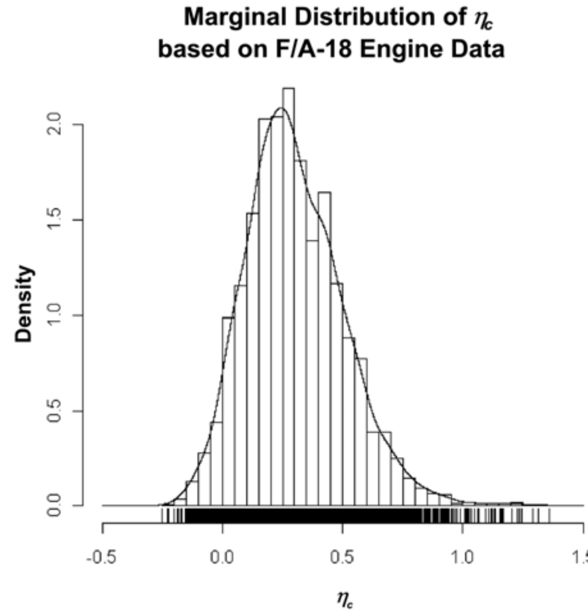

Figure 6 – The marginal distribution of the 10,000 MCMC samples of η_c obtained from the F/A-18 engine data indicates only a small risk of engine *critical life* reductions with repeated repair/refurbishment.

For F/A-18 engine *critical life* to reduce with repeated repair/refurbishment, it is necessary for $\eta_c < 0$. Based on the marginal samples of η_c, as reflected in figure 6, obtained by using the procedure described in section 2 with the data in figure 3 and Table 4, there is less than a 5% risk that F/A-18 engine *critical life* will reduce with repeated repair/refurbishment.

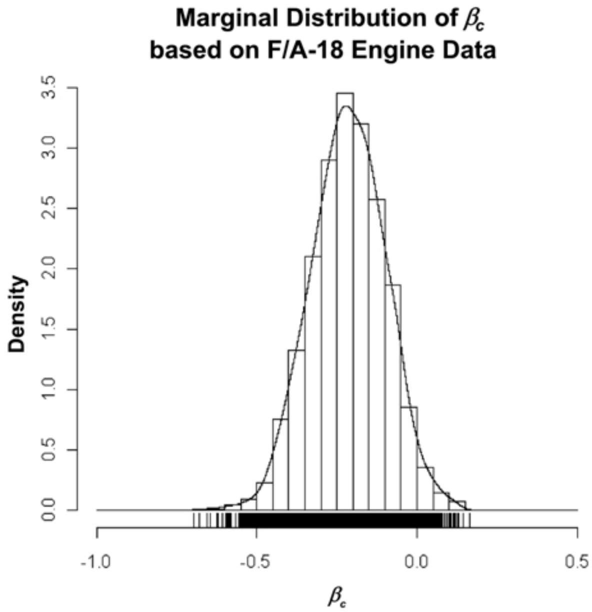

Figure 7 – The marginal distribution of the 10,000 MCMC samples of β_c obtained from the F/A-18 engine data indicates that there is a substantial risk of the engine failure mode trending towards *infant mortality* with repeated repair/refurbishment.

For the F/A-18 engine failure mode to trend toward *infant mortality* with repeated repair/refurbishment, $\beta_c < 0$. Based on the marginal samples of β_c, as reflected in figure 6, obtained by using the procedure described in section 2 with the data in figure 3 and Table 4, there is better than 97% probability that F/A-18 engine failure mode trends toward *infant mortality* with repeated repair/refurbishment. For most repair or maintenance technicians, this result agrees with intuition that repeated repair/refurbishment cycles is more likely to lead to earlier failures.

Results Derived from the MCMC Sampling

The previous subsection of this report identified the insights regarding the effects of repeated repair/refurbishments on the reliability of the F/A-18 engines that may be developed directly from the MCMC samples of η_c, and β_c. How $\eta(N_r)$ and $\beta(N_r)$ change as a function of N_r may provide additional insights. Samples of $\eta(N_r)$ and $\beta(N_r)$ may be developed by merely evaluating equations (3) at the MCMC samples of η_0, β_0, η_c, and β_c. This provides the uncertainty distributions for *critical life* and failure mode as a function of N_r given the data. Figures 8 and 9 best display these distributions using modified barcharts for $\eta(N_r)$ and $\beta(N_r)$ respectively. The bars in figures 8 and 9 start on the left at the 5th quantile for the distribution, and end at the 95th quantile on the right. Based on the data, there is 90% certainty that the true value of the quantity in question lies somewhere on the bar. The gray vertical lines on the bars are the modes of the distributions, and the color density on the bar is directly proportional to the probability density. These modified barcharts provide insights into these distributions visually, and are particularly useful for comparisons, especially of risk assessments such as presented in this report. Note in figures 8 and 9 that abscissa is logarithmically scaled.

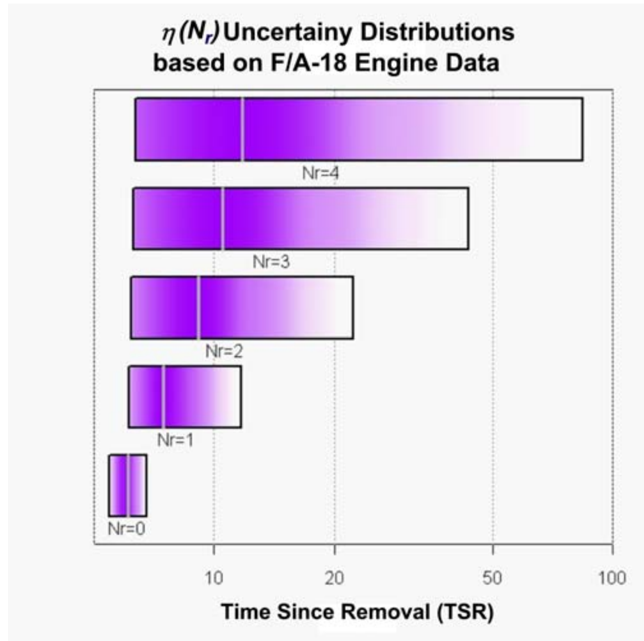

Figure 8 – Based on the F/A-18 engine data, *critical life* becomes more uncertain with repeated repair/refurbishment, yet trends towards larger values as it becomes more uncertain.

Figure 8 shows that the *critical life* uncertainty grows significantly (width of the bars grows) with repeated repair/refurbishment cycles. However, it is very interesting to note that most of this increase in uncertainty is gained to the right, towards higher values of *critical life*. Note that even for $N_r = 4$, the mode is still far to the left side of the distribution. The mode location however does move monotonically to the right as N_r increases, but nowhere near as fast as the 95th quantile. The lighter color densities to the right as N_r increases show that there is very little certainty based on the data that the true *critical life* reaches these larger values. However, for the bars in figure 8 with smaller values of N_r, these larger values of *critical life* have essentially no probability of being true. This is not an intuitive result in that most repair or maintenance technicians might expect earlier failures to be more likely with repeated repair/refurbishment cycles. Recall that the *critical life* is the life at which 63.2% of all failures will have occurred.

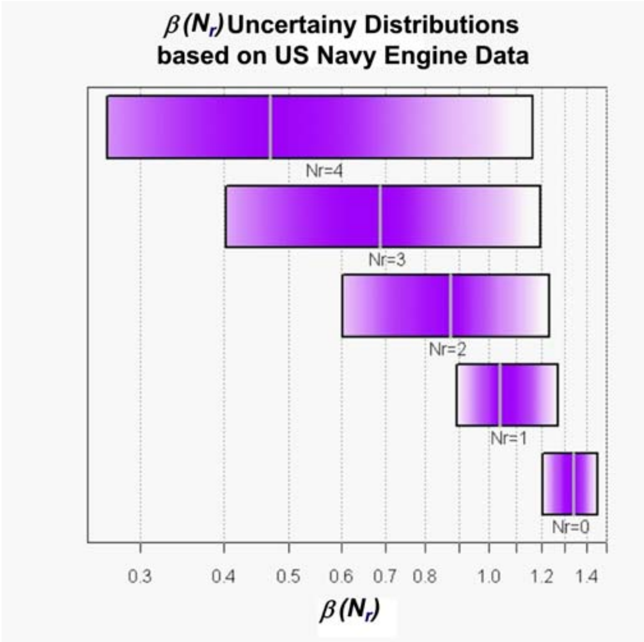

Figure 9 – Based on the F/A-18 engine data, the failure mode becomes more uncertain and trends towards *infant mortality* with repeated repair/refurbishment.

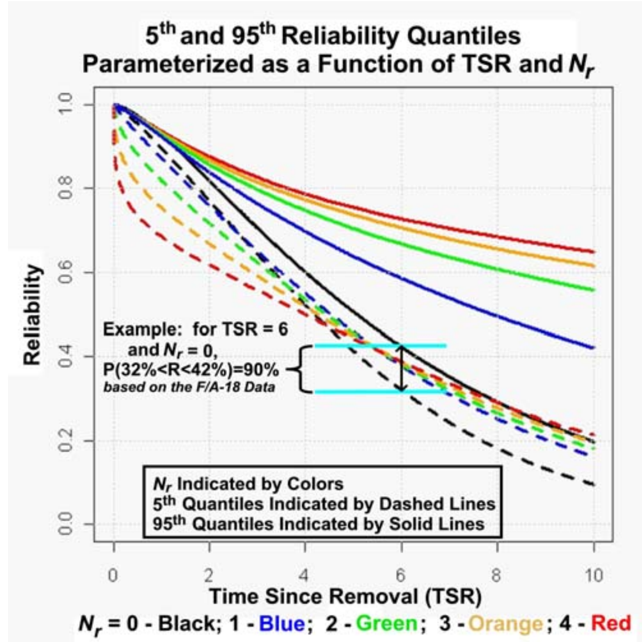

Figure 10 – Based on the F/A-18 engine data, reliability uncertainties vary non-intuitively with repeated repair/refurbishment.

In figure 9, the mode of $\beta(N_r)$ based on the data remains close to the center of each bar. As for the distributions of $\eta(N_r)$ in figure 8, uncertainty for $\beta(N_r)$ grows as N_r increases, however for $\beta(N_r)$ the modes shift more radically than they did for $\eta(N_r)$. Between figures 8 and 9, it becomes easy to conclude with significant certainty based on the data that repair/refurbishment does not return the F/A-18 engines to a new condition. If the bars in each of figures 8 and 9 overlaid each other better, with mode values very close, then this conclusion might be suspect.

The best way to display the reliability uncertainty models is to parameterize specific quantiles of reliabilities obtainable from equation (12) as a function of both TSR and N_r. Figure 10 presents these parameterizations for the 5^{th} and 95^{th} quantiles for reliabilities for the F/A-18 engines based on the data as a function of N_r.

Figure 10 uses dashed lines to indicate the 5^{th} quantile parametric reliability, and solid lines to indicate the 95^{th} quantile parametric reliability. Figure 10 also uses color to indicate the value of N_r for the parameterized quantiles: **black** for $N_r = 0$, **blue** for $N_r = 1$, **green** for $N_r = 2$, **orange** for $N_r = 3$, and **red** for $N_r = 4$. A vertical slice in figure 10 reveals that there is a 90% probability based on the F/A-18 engine data that the reliability is between 32% and 42% at TSR = 6 if there have been no repair/refurbishments (for $N_r = 0$, **black** dashed line up to **black** solid line).

The reliability uncertainty quantiles in figure 10 indicate that reliability for different values N_r as a function of TSR does not behave linearly, or even intuitively. Figure 10 is very information and insight rich. The 95^{th} quantile reliabilities increase as the number of repair/refurbishments increases, and even more so as TSR increases. Out beyond TSR = 5.8, the 5^{th} quantile reliabilities also increase as the number of repair/refurbishments increases. At TSR = 10, based on the solid black line, the F/A-18 engine data reveals that there is a 95% probability that the true reliability given no repair/refurbishments ($N_r = 0$) is less than 20%. Strangely enough, the very same data reveals via the dashed red line at TSR = 10, that there is a 95% probability that the true reliability is better than 20% after four repair/refurbishments cycles ($N_r = 4$). This means that based on the F/A-18 engine data, repeated repair/refurbishment appears to actually improve reliability dramatically for very large values of TSR, with 95% certainty. This rather startling result must however be tempered by the fact that none of the F/A-18 engine data had failures or survivors beyond TSR = 7.3, and that the reliability levels at large values of TSR are probably considerably below required levels. However, this remains a good example of the non-intuitive insights obtainable by taking advantage of the covariate information associated with the F/A-18 engine data, only possible using the covariate Weibull model and the conditional inferential method presented in section 2.

Hypothetical Preventative Maintenance Planning Using these Results

How this information can be used in preventative maintenance planning is best described using a hypothetical example. The example that follows is not intended to be representative of actual F/A 18 characteristics and performance. This example demonstrates that the insights gained from use of the covariate Weibull model in conjunction with the conditional inferential method presented in section 2 may be used to improve on a fixed periodic schedule of engine removals.

Suppose the F/A-18 had a required 70% mission assurance for a mission length of 0.5 in TSR units. For purposes of this hypothetical example, the engine is the only part that will ever fail; all other systems for this aircraft are presumed perfect. That 70% mission assurance requirement immediately translates to a minimum acceptable reliability for this engine of 70%. Suppose also that the maximum acceptable risk for the engine not achieving this reliability is only 5%. Suppose also that a periodic schedule of engine removals for FOD inspections, and if needed repair/refurbishments, is fixed at intervals of 1.2 in TSR units.

The information in figure 10 can be used to plan a preventative maintenance schedule that takes advantage of the covariate information N_r that guarantees satisfaction of this mission assurance requirement at the maximum acceptable risk. Figure 11 is an expansion of figure 10 that will demonstrate more easily how this information may be used to accomplish this.

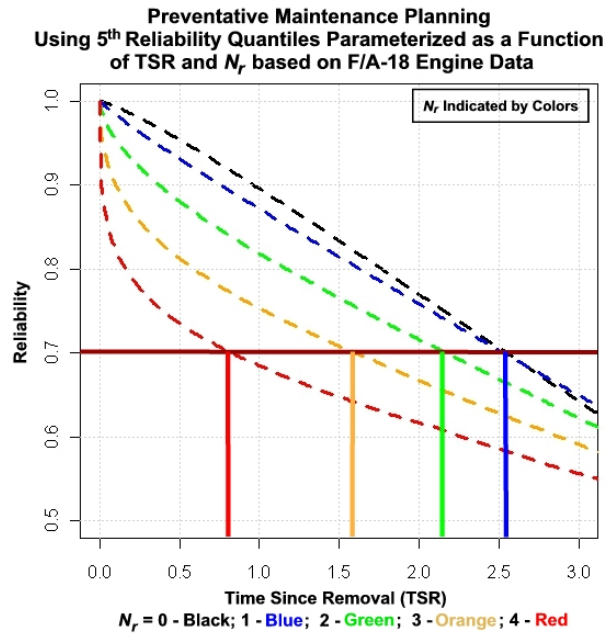

Figure 11 – Preventative maintenance planning that takes advantage of the covariate number of repair/refurbishments may be performed using an expansion of figure 10 using only 5th quantiles.

In figure 11, where the dashed lines cross the required 70% reliability level (brown horizontal line) provides some very useful information for this hypothetical example. Before and at these junctures, there is no more than a 5% risk that the reliability will be below 70% for the respective values of N_r. The **black** and **blue** dashed lines both (pure coincidence for this engine and hypothetical example) cross the required 70% reliability line at TSR = 2.55 (**blue** vertical line). If the F/A-18 engine has repaired/refurbished no more than once, the reliability requirement is satisfied for a mission duration of 0.5 in TSR units at less than the maximum acceptable risk for engine life up to TSR = 2.55. It would be very reasonable to schedule preventative maintenance no later than TSR = 2.05 (TSR = 2.55 less the mission duration of 0.5) for engines that have only been repaired/refurbished no more than once. This value is much larger than the fixed interval of 1.2 in TSR units, and cost savings and operational efficiencies may be possible using it instead.

By examining the other respective colored dashed and vertical lines in like fashion, figure 11 reveals that preventative maintenance at TSR = 1.65 for $N_r = 2$ (TSR = 2.15 less the mission duration of 0.5), at TSR = 1.1 for $N_r = 3$ (TSR = 1.6 less the mission duration of 0.5), and at TSR = 0.33 for $N_r = 4$ (TSR = 0.83 less the mission duration of 0.5) all similarly satisfy the hypothetical F/A-18 engine reliability requirement at less than the maximum acceptable risk. For repair/refurbishment cycle numbers of $N_r < 3$, these proposed inspection scheduled times are greater than the current fixed inspection interval of 1.2 in TSR units, and cost savings and operational efficiencies may be possible. After three or more repair/refurbishment cycles, the required reliability at the maximum acceptable risk is not achievable with a fixed inspection schedule interval of 1.2 in TSR units.

At some point, considering the costs of preventative maintenance versus the costs of a new engine and it makes sense that an engine could be retired from service; it is less expensive to replace the engine than to repair/refurbish it for very short mission times. In this example, after the fourth repair/refurbishment, the preventative maintenance interval that will meet the reliability requirement at the maximum acceptable risk is less than the hypothetical mission duration. After four repair/refurbishments, it is not possible to satisfy the mission assurance requirement for the specified mission duration of 0.5 in TSR units at the required maximum acceptable risk.

6. CONCLUSIONS

There are several important conclusions from the work presented in this report.

First, the covariate Weibull model used in conjunction with the conditional inferential method presented in section 2 can be used to effectively investigate whether repeated repair/refurbishments improve or degrade an aerospace system's reliability, even when the data is dominated by survivors. Note that the conditional inferential method presented in section 2 avoided many questionable assumptions.

Second, by using the covariate Weibull model and conditional inferential method presented in section 2 for the data presented in section 4, the authors discovered that there is less than a 5% risk that *critical life* decreases with repeated repair/refurbishment for the F/A-18 engines. This rather strong confirmation that *critical life* increases for the F/A-18 engines with repeated repair/refurbishment is non-intuitive, and may offer opportunities to improve preventative and corrective maintenance planning.

Third, by using the covariate Weibull model and conditional inferential method presented in section 2 for the data presented in section 4, there is a 97% probability that the failure mode moves towards *infant mortality* (decreasing β) with repeated repair/refurbishment. This result confirms normal intuition about repairs, that earlier failures should be expected after repair and return to service. The insight that reliability of the F/A-18 engines also improved as a function of repeated repair/refurbishments for large values of TSR was entirely non-intuitive.

Fourth, based on the hypothetical example presented in section 5, improvements in availability and mission assurance for these F/A-18 engines may be achieved through modifications of preventative maintenance schedules for engine removals based on numbers of repair/refurbishments. This result is of significant operational importance as the periodic inspection used to detect FOD requires a high level of maintainer knowledge, skills, and time. Reducing the frequency of repair/refurbishments relieves this workload burden and potentially the number of personnel deployed in harm's way at sea and in warzones.

Fifth, based on the hypothetical example presented in section 5, improvements in preventative maintenance cost effectiveness may be possible using the MCMC samples presented in this report in conjunction with the methods developed by Powell [17] for finding optimal cost preventative maintenance intervals.

Sixth, the statistical analysis and unanticipated results obtained by using the covariate Weibull model and conditional inferential method presented in this report justifies collection of additional maintenance data and further engineering analysis to gain a better understanding of the drivers of this pattern of reliability.

Seventh, the covariate Weibull model and conditional inferential method presented, validated, and exercised in this report may be used to investigate potential improvements in availability, mission assurance, and cost effectiveness of preventative and corrective maintenance for any aerospace system that is repeatedly repaired/refurbished and returned to service.

Further investigations of the method presented in this report using the covariate Weibull model and conditional inferential approach remain. The method should be investigated via sensitivity analyses regarding proportions of survivor to failure data to determine just how many failure data are actually needed to obtain useful results. Via simulated data, various values of η_0, β_0, η_c, and β_c should be explored to bound the detectability of effects that could be used to improve preventative and corrective maintenance. These and similar results should be investigated as to other measures related to the coupling of reliability, availability, maintainability, and logistics to determine if other improvements are possible.

REFERENCES

[1] R. C. Millar, T. A. Mazzuchi and S. Sarkani, "Nonparametric Statistical Analysis of the Reliability of a Naval Aviation Propulsion System" *Naval Engineers Journal*, vol. 121, no. 2, pp. 111-121, 2009

[2] R. C., Millar, T. A. Mazzuchi, and S. Sarkani, "Application of Non-Parametric Statistical Methods to Reliability Database Analysis", SAE AeroTech, (Paper 2007-01-3861) Los Angeles, 2007

[3] R.C. Millar, Non-Parametric Analysis of a Complex Propulsion System Data Base (Dissertation), The George Washington University, 2007

[4] Wallodi Weibull, "A Statistical Distribution Function of Wide Applicability," ASME Journal of Applied Mechanics, Transactions of the American Society Of Mechanical Engineers 73, 293-297, September 1951.

[5] Wallodi Weibull, Discussion, ASME Journal of Applied Mechanics, Transactions of the American Society Of Mechanical Engineers, 233-234, June 1952.

[6] Peter Congdon, Applied Bayesian Modelling, John Wiley & Sons Ltd., West Sussex, 2003.

[7] Joseph G. Ibrahim, Ming-Hui Chen, and Debajyoti Sinha, Bayesian Survival Analysis, Springer Science+Business Media, Inc., New York, 2001.

[8] D.J. Lunn, A. Thomas, N. Best, and D. Spiegelhalter, WinBUGS -- a Bayesian modelling framework: concepts, structure, and extensibility, Statistics and Computing, 10, 325-337, 2000.

[9] C. L. Smith, J. Knudsen, and T. Wood, Advanced SAPHIRE -- Modeling Methods for Probabilistic Risk Assessment via the Systems Analysis Program for Hands-On Integrated Reliability Evaluations (SAPHIRE) Software, May 2009.

[10] Harold Jeffreys, Theory of Probability. Oxford University Press, Oxford, 1939.

[11] James O. Berger, Statistical Decision Theory and Bayesian Analysis, Springer-Verlag, New York, 1980.

[12] D. S. Sivia, Data Analysis, A Bayesian Tutorial, Oxford University Press, Oxford, 1996.

[13] Samuel A. Schmitt, Measuring Uncertainty, An Elementary Introduction to Bayesian Statistics, Addison-Wesley Publishing Company, Inc., Philippines, 1969.

[14] Mark A. Powell, "Risk Assessment Sensitivities for Very Low Probability Events with Severe Consequences," Proceedings from the 2010 IEEE Aerospace Conference, Big Sky, MT, March 5-12, 2010.

[15] Christian P. Robert and George Casella, Monte Carlo Statistical Methods, Springer-Verlag, New York, 1999.

[16] W. R. Gilks, S. Richardson, and D. J. Spiegelhalter, Markov Chain Monte Carlo in Practice, Chapman & Hall, Boca Raton, 1996.

[17] Mark A. Powell, "Optimal Cost Preventative Maintenance Scheduling for High Reliability Aerospace Systems," Proceedings from the 2010 IEEE Aerospace Conference, Big Sky, MT, March 5-12, 2010.

BIOGRAPHY

Mark Powell has practiced Systems Engineering for over 35 years in a wide range of technical environments including DoD, NASA, DOE, and commercial. More than 25 of those years have been in the aerospace arena. His roles in these environments have included project manager, engineering manager, chief systems engineer, and research scientist. His academic affiliations have included the University of Idaho, Stevens Institute of Technology, and the University of Houston, Clear Lake. Mr. Powell maintains an active engineering and management consulting practice throughout North America, Europe, and Asia. Beyond consulting, he is sought frequently as a symposium and conference speaker and for training, workshops, and tutorials on various topics in Systems Engineering, Project Management, and Risk Management. Mr. Powell is an active member of AIAA, Sigma Xi, the International Society for Bayesian Analysis, and the International Council on Systems Engineering, where he has served as Risk Management Working Group Chair and as Assistant Director for Systems Engineering Processes.

Richard Millar is an Associate Professor with the Naval Postgraduate School Department of Systems Engineering, based at the Patuxent River Naval Air Station. He has 35+ years experience in the design and development of gas turbine engines and their integration with aircraft propulsion & power systems. He has worked in this field at General Electric, United Technologies, Rolls-Royce, Boeing, Lockheed Martin and BAE Systems prior to joining NAVAIR in 2003.

Dr. Millar has an active research program currently focused on the systems engineering and development of integrated instrumentation / sensor systems for use in the test & evaluation, CBM & PHM and control of aerospace equipment.