Method for Detection and Confirmation of Multiple Failure Modes with Numerous Survivor Data

Mark A. Powell
Attwater Consulting
P.O. Box 57702
Webster, TX 77598-7702
208-521-2941
attwater@aol.com

Abstract—Modern aerospace systems are often subject to multiple modes of failure. [1] [2] Unless the failure modes have radically different statistical characteristics, it can be very difficult to recognize in failure data that multiple failure modes are in play. When using graphical inferential methods for reliability analysis, the potential for multiple failure modes may be suggested by a *bent* regression line. Subsequent analysis of these modes is usually performed by segregating the data to separate sets to be processed separately for each failure mode, usually segregated about the *bend* in the regression line. This data segregation process raises a number of questions, especially when numerous survivor data are to be processed. The primary issue with a survivor datum in this case is that it reflects that the system has not yet failed, in any mode. Assigning a survivor datum to any particular failure mode data set for separate processing appears completely arbitrary, and may produce spurious results that may then result in unnecessary costs or unacceptable risks in preventative maintenance.

The US Coast Guard encountered such a dilemma with failure data for the AC generator subsystem for the HU-25 aircraft. Their data consisted of 45 failures and 41 survivors. When processed using the graphical inferential method provided by a commercially available reliability analysis tool, the results presented a *bent* regression line suggesting that two failure modes may be in play. Further analysis of these failure modes, via segregation of both the failure and survivor data at the *bend* and reprocessing of the segregated data sets, was neither convincing nor comforting to the US Coast Guard.

This report investigates the use of a mixture model to represent potential multiple failure modes. A method is developed to use this model using a conditional inferential approach and is used to detect and confirm multiple failure modes, without any failure and survivor data segregation. This model and method are validated using simulated data sets generated with a variety of dual failure modes, both easy and difficult to discern visually in the data, and demonstrates that multiple failure modes can be reliably detected when the failure modes are represented in the data by proportions between 25 and 75%, even when the multiplicity of the modes is not visually obvious in the data. The HU-25 AC generator data are then processed using this method and the results demonstrate that the suspected two distinct failure modes are indeed in play.

The model and method presented in this report can be used for any aerospace system which from the data is suspected of having multiple modes of failure.

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1. INTRODUCTION

Aerospace systems are generally designed and built to meet very stringent performance and reliability requirements. Considering the complexity of modern aerospace systems, it is quite often very difficult to ascertain why a particular system or subsystem might fail, or if multiple causes for failure are present in sets of failure data. Complicating matters, for these systems that might be suspected of failing for multiple reasons, due to the stringent reliability requirements to which they were designed and built, significant numbers of survivor or suspension data may be available as well. A survivor datum is where the system has been observed to perform for some period without failing. Survivor data provide very valuable information regarding the reliability of a system, yet no information that can relate to any particular failure mechanism or mode. In reliability and risk analyses, survivor data may actually dominate data sets [1].

There are a number of software tools available to perform reliability analysis. Most focus on use of the Weibull

[1] 978-1-4244-7351-9/11/$26.00 ©2011 IEEE.
[2] IEEEAC paper #1497, Version 4, January 17,2011

distribution model and use classical graphical inferential methods to infer point estimates of the parameters of the Weibull model. This method is referred to as *Median Rank Regression*, and is capable of incorporating survivor data in the statistical inference. These tools often highlight the potential for multiple modes of failure in the data when a *bent* regression line best fits the data. The typical procedure used when a *bent* regression line presents is to segregate the data at the bend, and to reprocess the segregated data sets separately to infer the parameters of the Weibull model for each failure mode. This data segregation and reprocess procedure raises some questions, particularly when there are appreciable numbers of survivor data. Recall for a survivor datum, the system has not failed for any reason, or as a result of any failure mode. How then is any particular survivor datum assigned to any set of failure data as a valid part of that set? How would assignment of a particular survivor datum affect any of the failure modes inferences?

The US Coast Guard used such a commercially available reliability analysis tool that uses graphical inferential methods [2] for a set of failure and survivor data for the AC generator used in the HU-25 Guardian aircraft. Figure 1 shows this aircraft.

Figure 1 – The US Coast Guard HU-25 Guardian aircraft AC generator was suspected of exhibiting multiple modes of failure.

The AC generator for the HU-25 Guardian was experiencing failures that were unexpected. The failure data collected for the AC generator consisted of 45 failures (four at initial start), and 41 survivors. When these data were processed (omitting the zero time failures) using a commercially available reliability analysis software tool, the results presented a *bent* regression line, suggesting that multiple failure modes might be in play. Figure 2 is the plot produced by this tool.

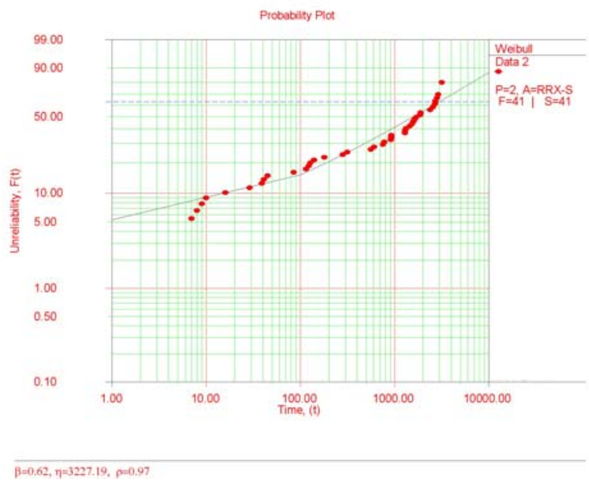

Figure 2 – Processing of the failure and survivor data for the HU-25 AC generator using a commercially available reliability analysis tool suggested multiple modes of failure via the *bent* regression line.

The plot in figure 2 reveals many features. First, the four zero time failures were not included in the data set since graphical inferential methods cannot use them. In the lower left corner, the inferred point estimates of the Weibull parameters for the entire data set indicated that the failure mode was *infant mortality* (shape parameter $\beta = 0.62$). A *bent* regression line is presented, suggesting that multiple failure modes may exist in the data.

The US Coast Guard analysts went on to segregate the failure and survivor data at various points in time and reprocess the segregated sets of data separately. The results of each segregation and reprocessing were not particularly comforting nor convincing to the analysts. Graphical inferences from some segregation sets of data confirmed the *infant mortality* mode (inferred point estimate β values ranging from 0.47 to 0.78) as inferred from the entire data set, and others indicated *old age* failure modes (inferred point estimate β values ranging from 4.67 to $\beta > 11$). That two failure modes seemed to be confirmed by the data segregation and reprocessing was not in question. The failure modes inferred, and the specific point estimate values of the shape parameter β, however, do play a significant role in investigation of the cause of the failures and how to develop a cost effective preventative maintenance schedule to assure aircraft availability.

The author of this report had previously worked with the US Coast Guard analysts working with this problem on finding the optimal cost preventative maintenance interval for a cooling turbine for the C130 aircraft [3]. The analysts thus presented these graphical inferential method results to the author of this report for comment. The author of this report suggested that perhaps a different approach might yield additional insights.

This different approach started with a mixture of two Weibull distribution models. If there were truly two modes of failure in play in the HU-25 AC generator data set, then some of these data should have come from one, and some from the other. A conditional inferential method was developed to infer the joint distribution of parameters from both Weibull models along with the mixture parameter. This method employed models that reflect objectivity for all parameters, for both Weibull models' parameters and for the mixture ratio parameter, to avoid any influences favoring any particular failure mode, or that multiple modes exist.

Because of the analytical complexity of this mixture model formulation, coupled with the complexity of the conditional inferential methods employed, numerical methods (Markov Chain Monte Carlo) were required to obtain samples of the joint distributions of all parameters. From the marginal and paired joint distributions of these samples, convincing arguments can be developed as to whether multiple modes of failure exist in the data or not, as well as characteristics of those modes.

Before using the mixture model and conditional inferential approach to investigate whether multiple failure modes exist in the HU-25 data, investigatory sets of simulated failure and survivor data were developed for various mixture ratios, for distinct failure modes (one *infant mortality* $\beta = 0.7$, the other *early wearout* $\beta = 3.7$). These data were processed to validate that the method could indeed detect and confirm that multiple modes of failure were in play in the data for mixture ratios between 25% and 75%, and could not detect nor confirm multiple modes when the mixture ratios were more exaggerated.

The AC generator data for the HU-25 aircraft were then processed using this mixture model and conditional inferential and numerical methods, and results were obtained. This approach inferred that two failure modes did indeed exist in the AC generator data, and that the mixture ratio was most likely ~ 41%. One failure mode was most likely *infant mortality* with $\beta \sim 0.3$, and that the other was most likely *early wearout* with $\beta \sim 2$. These inferred results were dramatically different from those obtained using the graphical inferential methods, with any segregation sets of data. Examination of the parameter samplings was comforting and convincing.

This report presents the development of the mixture model and conditional inferential and numerical methods that may be used to investigate multiple failure modes in sets of data that include failures and survivors. It presents preliminary validation demonstrations using a suitable set of simulated data that the method can reliably detect and confirm multiple failure modes in sets of data that include failures and survivors, for distinctly different failure modes with mixture proportions in the data that would either present a bent regression line using a graphical inferential method, or perhaps present visible multiple modes in scatterplots of the data.

This report then presents the results from processing the HU-25 AC generator data using the mixture model and conditional inferential method. Two distinct modes were found using this model and method, that were quite different from those obtained from any data segregations and processing using the commercially available reliability tool.

2. METHOD

Development of the method to detect and confirm multiple failure modes for failure data that include numerous survivor data begins with development of a mixture model, and proceeds with development of a conditional inferential method to process the data. Due to the complexity of both the mixture model and conditional inferential method, numerical methods must be employed to obtain samplings of the joint distribution model of all parameters.

Mixture Model Development

Equation (1) provides the general Weibull density function, which has a location parameter $t_1 \geq 0$, a scale parameter $\eta > 0$, and a shape parameter $\beta > 0$.

$$pd(t_f \mid t_1, \eta, \beta) = \left(\frac{\beta}{\eta}\right)\left(\frac{t_f - t_1}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_f - t_1}{\eta}\right)^{\beta}} \qquad (1)$$

The Weibull model is a very general model for reliability related problems in that the parameters all have physical meanings. This is not the case for many probability distribution models. The location parameter t_1 represents the time before which failures cannot occur, and is called the *failure-free time*. The scale parameter η is the time at which 63.2% of all failures will have occurred, and is called the *critical life*. The shape parameter β is an indicator of failure mode. Values of $\beta < 1$ indicate an *infant mortality* failure mode. Values of $\beta = 1$ indicate a *useful life* failure mode. Values $1 < \beta < 4$ indicate an *early wearout* failure mode. And, values of $\beta > 4$ indicate an *old age* failure mode. Depending on the values of these parameters, the Weibull model can represent just about any uni-modal, one-sided distribution shape for failures imaginable, with skews to either left or right.

An important aside relative to this density formulation: Weibull's original paper [4] published in September 1951 provided a distribution function that would produce the density function in equation (2).

$$pd(t_f \mid t_1, \lambda, \alpha) = \left(\frac{\alpha}{\lambda}\right)\left(t_f - t_1\right)^{\alpha-1} e^{-\frac{\left(t_f - t_1\right)^{\alpha}}{\lambda}} \tag{2}$$

In discussions of Weibull's paper [5] published in June 1952, Weibull noted that his distribution function as originally published was incorrect by stating that the "…parentheses are an awkward misprint." Correction of this misprint produces the density function in equation (1).

The significance of this typographical error is profound. First, equation (2) cannot be reparameterized to produce equation (1) without comingling the parameters; the density function in equation (2) is fundamentally flawed since neither λ nor α can be classed as proper location, scale, or shape parameters. Second, textbooks [6] [7] exist that use the incorrect density function in equation (2) for the Weibull model. And, third, there are statistical software packages and tools [8] [9] that use the incorrect density function in equation (2) for the Weibull model. The caveat for the reader of this report is that whenever encountering any work using the Weibull model, and when considering any software package or tool, it is imperative to verify that the implementation of the Weibull model uses the proper form expressible as equation (1). The results obtained in any analytical work or through use of a software package that uses a form expressible as equation (2) may be pathological.

For the work presented in this report, the location parameter t_1 in equation (1) is set to zero. There exists no reason to believe that any HU-25 AC generator could not fail the instant operation begins.

A mixture model to represent the uncertainty for failures to be observed where two failure modes exist can be developed by using two Weibull models and introducing a new parameter to represent the mixture proportion. One of the Weibull models will have parameters η_a and β_a, and the other will have η_b, and β_b. The mixture proportion parameter (proportion of failures from the Weibull model with parameters η_a and β_a) will be represented by γ. Equation 3 presents this mixture model where $Weib_a(t_f|\eta_a, \beta_a)$ and $Weib_b(t_f|\eta_a, \beta_a)$ are the two Weibull distribution models.

$$\begin{aligned} pd(t_f \mid \gamma, \eta_a, \beta_a, \eta_b, \beta_b) &= \gamma * Weib_a\left(t_f \mid \eta_a, \beta_a\right) \\ &\quad + (1-\gamma) * Weib_b\left(t_f \mid \eta_b, \beta_b\right) \\ &= \gamma * \left(\frac{\beta_a}{\eta_a}\right)\left(\frac{t_f}{\eta_a}\right)^{\beta_a - 1} e^{-\left(\frac{t_f}{\eta_a}\right)^{\beta_a}} \\ &\quad + (1-\gamma) * \left(\frac{\beta_b}{\eta_b}\right)\left(\frac{t_f}{\eta_b}\right)^{\beta_b - 1} e^{-\left(\frac{t_f}{\eta_b}\right)^{\beta_b}} \end{aligned} \tag{3}$$

Conditional Inferential Approach

With conditional inferential methods, the joint probability density model for the parameters of the mixture model is developed based solely on the data. With this joint density, it is possible to compute any probability that might be useful. To develop the joint density of γ, η_a, β_a, η_b, and β_b given the data, Bayes' Law [10] is employed per equation (4).

$$\begin{aligned} &pd\left(\gamma, \eta_a, \beta_a, \eta_b, \beta_b \mid data\right) \\ &\quad \propto pd\left(data \mid \gamma, \eta_a, \beta_a, \eta_b, \beta_b\right) \\ &\qquad * pd\left(\gamma, \eta_a, \beta_a, \eta_b, \beta_b\right) \end{aligned} \tag{4}$$

In equation (4), the first term to the right of the proportion, $pd(data|\gamma, \eta_a, \beta_a, \eta_b, \beta_b)$, is the *likelihood*. When the data is limited to only failure times, this is the same likelihood function used in calculating maximum likelihood estimates. The second term to the right of the proportion, $pd(\gamma, \eta_a, \beta_a, \eta_b, \beta_b)$, is the joint *prior* density for γ, η_a, β_a, η_b, and β_b. The joint *prior* density is selected to model the knowledge or ignorance of γ, η_a, β_a, η_b, and β_b before obtaining the data. The proportionality in the equation is insignificant; the proportionality constant can always be calculated by integrating over all values of γ, η_a, β_a, η_b, and β_b. The term on the left, $pd(\gamma, \eta_a, \beta_a, \eta_b, \beta_b|data)$, the joint density of γ, η_a, β_a, η_b, and β_b given the data, is called the joint *posterior* density.

Selection of the *prior* model for some problems can pose some difficulty. Some decision makers feel that using a priori knowledge of the parameters somehow prejudices the results, casting the pall of a rigged decision subject to second-guessing. Beyond that, for many uncertainty models that might be selected for the data for various problems, the parameters have no useful physical meaning, and thus no reason exists to have any a priori knowledge of them. To address both of these difficulties, it is possible to use a *prior* density model that imparts no a priori knowledge of the parameters. This is called using a *noninformative* or *ignorance prior* [11]. Use of *ignorance priors* establishes a basis of maximum objectivity for the decision, and alleviates the difficulty of dealing with any second-guessing. The joint *prior* density model is generally structured such that the parameters are independent. Using Weibull models, because η and β are scale and shape parameters respectively, Jeffrey's *priors* [12] are very suitable as the *ignorance priors* for η_a, β_a, η_b, and β_b and are presented in equations (5).

$$\begin{aligned} pd\left(\eta_a\right) \propto \frac{1}{\eta_a};\ \ pd\left(\beta_a\right) \propto \frac{1}{\beta_a} \\ pd\left(\eta_b\right) \propto \frac{1}{\eta_b};\ \ pd\left(\beta_b\right) \propto \frac{1}{\beta_b} \end{aligned} \tag{5}$$

The mixture parameter γ is a simple Bernoulli model, having a value [0,1]. The ignorance prior for a parameter with a Bernoulli model is a Beta model with both shape parameters set to ½, and is presented in equation 6 .

$$pd\left(\gamma \mid a,b\right) \propto \gamma^{a-1}\left(1-\gamma\right)^{b-1} \\ = \gamma^{-\frac{1}{2}}\left(1-\gamma\right)^{-\frac{1}{2}} \tag{6}$$

Now, given as data N_f failures and N_s survivors (times of good inspections or when some other unrelated failure occurred), the *posterior* density model is formed in equation (7) using the mixture model from equation (4) with t_{fi} being the time of the i^{th} failure, and t_{sj} being the time of the j^{th} survivor.

$$\begin{aligned} & pd\left(\gamma,\eta_a,\beta_a,\eta_b,\beta_b \mid data\right) \\ & \propto \left[\gamma * \prod_{i=1}^{N_f}\left(\frac{\beta_a}{\eta_a}\right)\left(\frac{t_{fi}}{\eta_a}\right)^{\beta_a-1} e^{-\left(\frac{t_{fi}}{\eta_a}\right)^{\beta_a}} \right. \\ & \qquad \left. +\left(1-\gamma\right) * \prod_{i=1}^{N_f}\left(\frac{\beta_b}{\eta_b}\right)\left(\frac{t_{fi}}{\eta_b}\right)^{\beta_b-1} e^{-\left(\frac{t_{fi}}{\eta_b}\right)^{\beta_b}}\right] \\ & \qquad * \left[\gamma * \prod_{j=1}^{N_s} e^{-\left(\frac{t_{sj}}{\eta_a}\right)^{\beta_a}} + \left(1-\gamma\right) * \prod_{j=1}^{N_s} e^{-\left(\frac{t_{sj}}{\eta_b}\right)^{\beta_b}}\right] \\ & \qquad * \left(\frac{1}{\eta_a}\right) * \left(\frac{1}{\beta_a}\right) * \left(\frac{1}{\eta_b}\right) * \left(\frac{1}{\beta_b}\right) * \gamma^{-\frac{1}{2}}\left(1-\gamma\right)^{-\frac{1}{2}} \end{aligned} \tag{7}$$

In equation (7), the first term to the right of the proportion in brackets ([·]) is the *likelihood* for the failure data, the second term in brackets ([·]) is the *likelihood* for the survivor data, and the two remaining terms are the Jeffrey's *priors* for η_a, β_a, η_b, β_b, and γ respectively. One very nice feature of conditional inferential methods apparent in equation (7) is that survivor data can be used directly via the likelihood [13]. Observed times at which a subsystem in service has not failed comprise very important information that should not be neglected in the posterior or in the decision. For some problems, the number of survivor data may exceed that for failure data, and there may be only survivor data and no failure data at all. Conditional inferential methods provide solutions for these data sets; such solutions are not possible using classical methods without employing assumptions that may be questionable.

Numerical Methods

The uncertainty distribution in equation (7) is not analytically integrable. The solution is to use numerical methods, namely Monte Carlo methods [14]. Monte Carlo methods are used widely for accurately approximating the evaluation of probability integrals. Quite often, risk problems such as the subject of this report are solvable only using Monte Carlo methods.

The central issue to evaluating equations (7) using Monte Carlo methods is to obtain a large number of samples of γ, η_a, β_a, η_b, and β_b from the joint *posterior* uncertainty model in equation (7). There exist no statistical software packages with built-in samplers for the joint *posterior* density function of equation (7). The remedy is to use Markov Chain Monte Carlo (MCMC) methods to sample this *posterior*. MCMC methods allow full range sampling of arbitrary distributions of any dimension given the formulation of the joint density [15]. With sufficient MCMC sampling of the joint *posterior* in equation (7), it is possible to compute very accurate approximations for almost any measure or statistic of interest.

3. VALIDATION

Before using the approach presented in section 2 for the HU-25 AC generator data, it is prudent to validate the approach by using artificial data generated using known values of η_a, β_a, η_b, and β_b for various values of γ. 10,000 failure and survivor data were developed each for five values of the mixture parameter γ (0.01, 0.25, 0.5, 0.75, and 0.99) using known values of η_a, β_a, η_b, and β_b. Table 1 contains the values of η_a, β_a, η_b, and β_b used for these five sets of validation data.

Table 1: Validation Model True Parameter Values

Parameter	Value
η_a	800
β_a	0.7
η_b	1500
β_b	3.7

The values in Table 1 represent two distinct failure models. Figure 3 shows the failure densities produced by these two models.

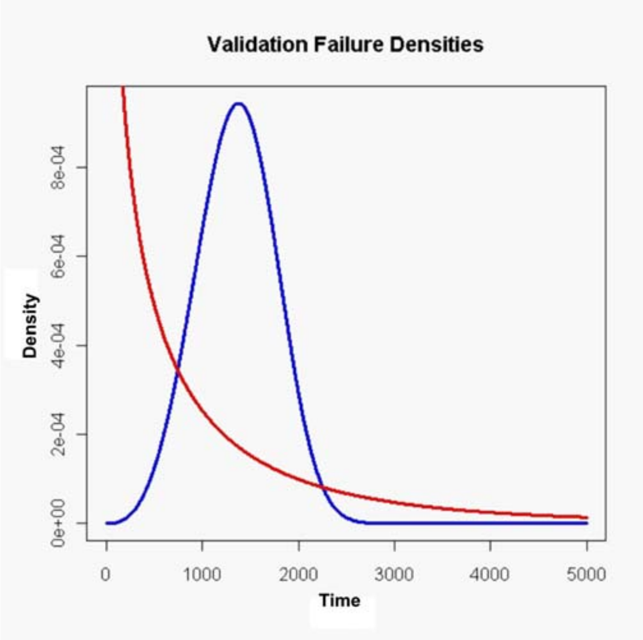

Figure 3 – The two validation failure models, *Weib$_a$* in RED and *Weib$_b$* in BLUE, will present multiple modes for some values of the mixture parameter, and not for others. Note that the densities overlap appreciably, and that *Weib$_a$* may present samples to the right of *Weib$_b$*.

Validation samples were generated using the following procedure. For each value of the mixture parameter γ, 10,000 uniform samples are obtained. For each uniform sample: if its value is less than the value of γ, then t_{samp} is selected from *Weib$_a$*; if not, then t_{samp} is selected from *Weib$_b$*. If t_{samp} is less than $t = 1{,}800$, then it is collected as a failure datum; if not, it is collected as a survivor datum at $t = 1{,}800$. Note in figure 3 that this survivor cutoff at $t = 1{,}800$ converts appreciable numbers of samples from failures to survivors for both failure models. The numbers of validation samples generated for the two Weibull models for the five values of the mixture parameter γ are presented Table 2.

Table 2: Validation Sample Numbers

γ	Validation Data Samples Numbers Generated			
	Weib$_a$		*Weib$_b$*	
	# t_f	# t_s	# t_f	# t_s
0.01	84	19	8463	1434
0.25	2100	435	6432	1033
0.5	4100	872	4334	694
0.75	6274	1262	2101	363
0.99	8170	1722	94	14

In Table 2, the survivor data represent about 15% of the total data. Figures 4-8 provide the samples and sample densities for validation failures for $\gamma = 0.01$, 0.25, 0.5, 0.75, and 0.99 respectively. Note that the failure samples are in the rug beneath the histogram and density curve in each figure, and that there is a distinct cutoff at $t = 1{,}800$ where the survivor data are set.

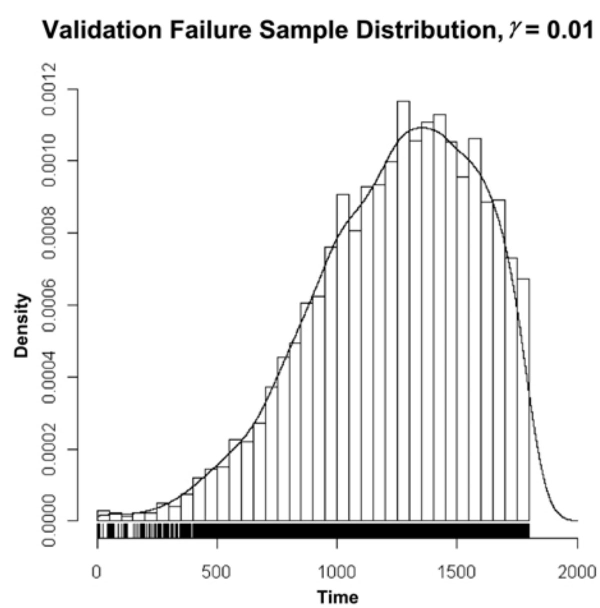

Figure 4 – The validation failure samples for $\gamma = 0.01$ show only a single failure mode.

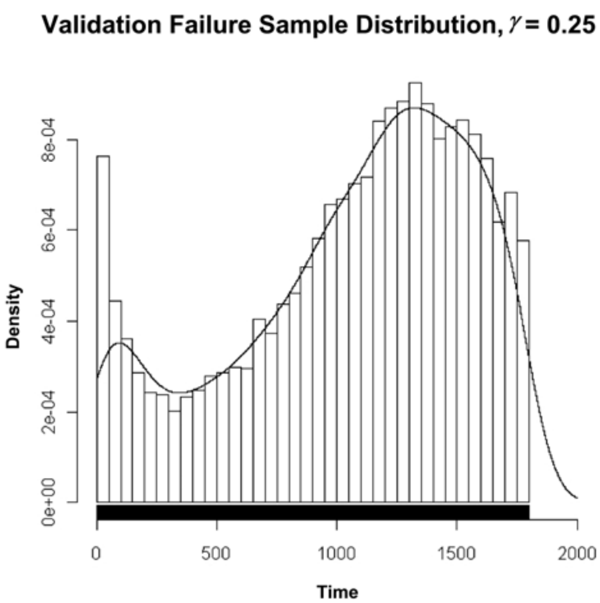

Figure 5 – Two failure modes are obvious in the validation failure samples for $\gamma = 0.25$.

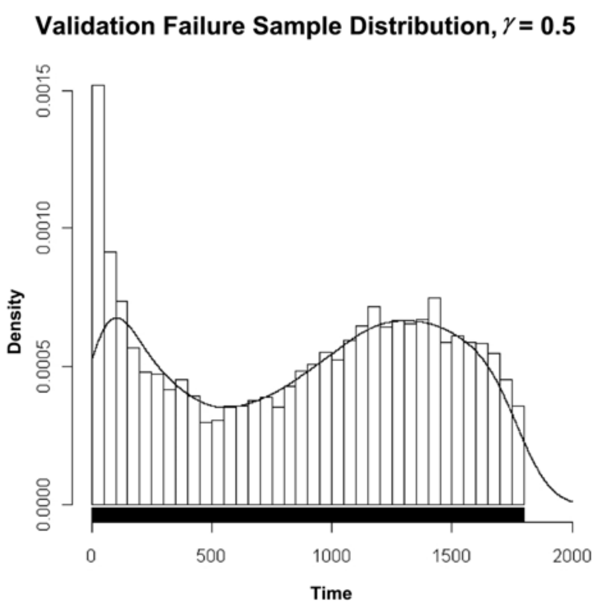

Figure 6 – Two failure modes are obvious in the validation failure samples for $\gamma = 0.5$.

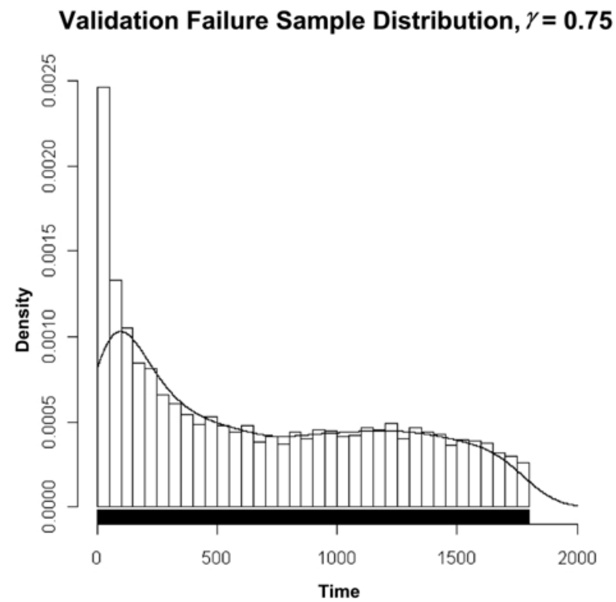

Figure 7 – It is diffucult to visually identify two failure modes in the validation failure samples for $\gamma = 0.75$.

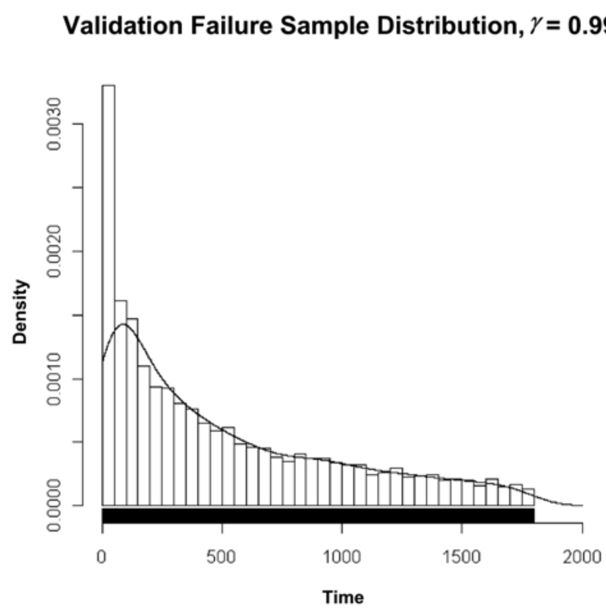

Figure 8 – The validation failure samples for $\gamma = 0.99$ show only a single failure mode.

Figures 5 and 6 ($\gamma = 0.25$ and $\gamma = 0.5$ respectively) show very strong indications of more than one failure mode. Figure 7 ($\gamma = 0.75$) hardly suggests that there may be more than one failure mode. Figures 4 and 8 ($\gamma = 0.01$ and $\gamma = 0.99$ respectively) do not in any way suggest that there may be multiple modes involved, which should not be surprising.

The procedure described in section 2 of this report was used to process each set of 10,000 failure and survivor validation data for the five values of γ. The Markov chains in each execution of the procedure were initialized to values of $\gamma = 0.5$, $\eta_a = \eta_b = 1{,}192$, and $\beta_a = \beta_b = 1.5$. By initializing at these values, the Markov Chains in the MCMC should only stabilize to multiple failure modes when the data upon which they are based contains different failure modes. For the validation data with $\gamma = 0.01$, the Markov chains did not ever stabilize for γ, η_a, or β_a. For the validation data with $\gamma = 0.99$, the Markov chains did not ever stabilize for γ, η_b, or β_b. These results are actually comforting. There were so few data generated for model $Weib_a$ when $\gamma = 0.01$, and for model $Weib_b$ when $\gamma = 0.99$, that stabilization of the Markov chains probably should not have been anticipated. As figures 4 and 8 demonstrate, the data for these rarely sampled models are not apparent, nor were they to the procedure. That the Markov chains did not stabilize when the data sampling densities showed no hint of multiple failure modes, instead of stabilizing on some phantom values, is a very good indication and validation that the procedure will not find failure modes not represented in the data.

For the other sets of validation data, those with $\gamma = 0.25$, 0.5, and 0.75, the Markov chains stabilized for all parameters. Table 3 provides marginal sampling statistics for these results for all parameters.

Table 3: MCMC Sampling Results for Validation Data for $\gamma = 0.25$, $\gamma = 0.5$, and $\gamma = 0.75$.

	$\gamma = 0.25$			
	Minimum	**Maximum**	**Mean (True)**	**σ**
γ	0.066	0.423	0.223 (0.25)	0.066
η_a	118.6	1864.4	777.5 (800)	351.2
β_a	0.573	1.156	0.789 (0.7)	0.077
η_b	1387.3	1593.4	1500.9 (1500)	28.9
β_b	2.822	4.892	3.638 (3.7)	0.295
	$\gamma = 0.5$			
γ	0.267	0.720	0.528 (0.5)	0.090
η_a	182.7	1223.7	747.1 (800)	218.6
β_a	0.579	0.922	0.707 (0.7)	0.043
η_b	1360.4	1657.8	1507.4 (1500)	52.3
β_b	2.359	5.632	3.893 (3.7)	0.599
	$\gamma = 0.75$			
γ	0.458	0.775	0.609 (0.75)	0.050
η_a	216.7	800.0	391.9 (800)	64.1
β_a	0.650	0.902	0.757 (0.7)	0.034
η_b	1333.7	1562.0	1454.9 (1500)	32.5
β_b	2.68	6.72	4.01 (3.7)	0.524

For values of $\gamma = 0.25$ and $\gamma = 0.5$ in Table 3, the mean values of the parameter samples agreed very well with the values used to generate the validation data sets. For these values of γ, that multiple failure modes existed were very obvious in the related figures 5 and 6. For $\gamma = 0.75$ in Table 3, the mean for the γ samples was only within 20% of the value used to generate the validation data. Also for $\gamma = 0.75$ in Table 3, the mean for the η_a samples was only within 50% of the value used to generate the validation data. The means for the remaining parameter samples for $\gamma = 0.75$ in Table 3, for β_a, η_b, and β_b, all agreed reasonably well with the values used to generate the validation data. Recall from figure 7 however, that it was difficult to determine visually from the density of the failure data that multiple failure modes were in effect. The primary factor in this less than ideal estimate was due to truncation of the validation failure samples at $t_{samp} = 1800$ to create the survivor data. This transformed over 14% of the *Weib*$_b$ failure samples into survivor samples, in a high probability density region as is observed in figure 3. Despite this artifact, the marginal density of the γ samples obtained from the validation failure and survivor data generated when $\gamma = 0.75$ strongly detected that multiple failure modes existed in the data, as shown in figure 9.

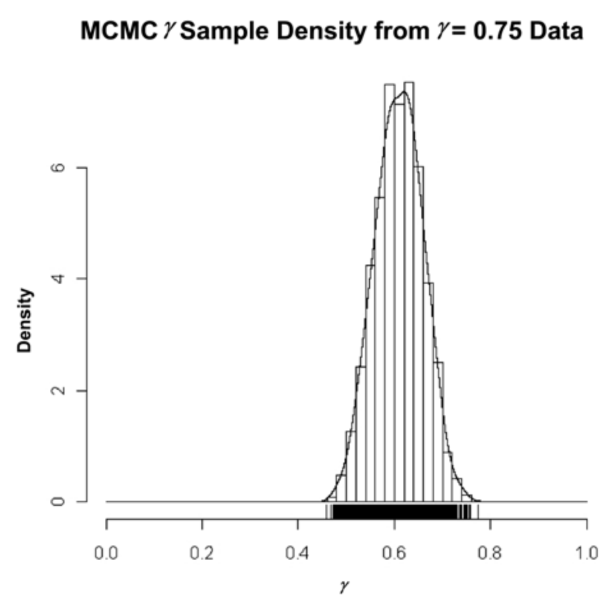

Figure 9 – The MCMC samples and sample density for γ based on the validation failure and survivor data set for $\gamma = 0.75$ reveals that despite the mean and mode not being very close to the true value, the sharpness of the MCMC sample density strongly suggests that multiple failure modes exist in the data.

Had the MCMC samples in figure 9 been spread over a much larger region, and the sample density been broad, perhaps then there would be some doubt about detection of multiple failure modes in the validation failure and survivor data.

4. DATA

The HU-25 AC generator failure and survivor data are presented in Figure 10.

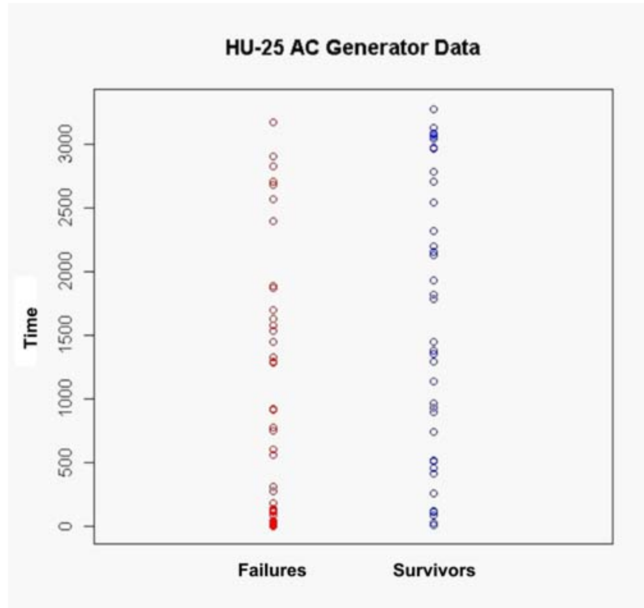

Figure 10 – The failure data for the HU-25 AC generator are displayed in the color RED. The survivor data are displayed in the color BLUE.

Visual inspection of the scatter of the failure data in figure 10 reveals no obvious multiple modes of failure.

5. RESULTS

The procedure described in section 2 was applied to the HU-25 AC generator failure and survivor data described in section 4 to obtain 10,000 joint samples of γ, η_a, β_a, η_b, and β_b from the joint *posterior* uncertainty model as in equation (7). The Markov chains were initialized to values of $\gamma = 0.5$, $\eta_a = \eta_b = 1{,}192$, and $\beta_a = \beta_b = 1.5$, and all Markov Chains stabilized. Table 4 presents the statistics for the marginal MCMC samples of these parameters.

Table 4: MCMC Sampling Statistics Obtained from the HU-25 AC Generator Data.

	Minimum	Maximum	Mean	σ
γ	0.026	0.869	0.412	0.148
η_a	0.7	3497.7	1106.1	987.0
β_a	0.033	0.574	0.284	0.068
η_b	1547.2	4496.2	3052.0	491.4
β_b	0.439	5.919	2.099	0.909

The sampling statistics in Table 4 were not as ideal as those in Table 3. The numbers of data used to were only 45 failures and 45 survivors, not the 10,000 data represented by the sampling statistics in Table 3. Examining the marginal MCMC sample densities for each parameter is instructive. Figures 11-15 provide these samples and marginal sample densities for the parameters.

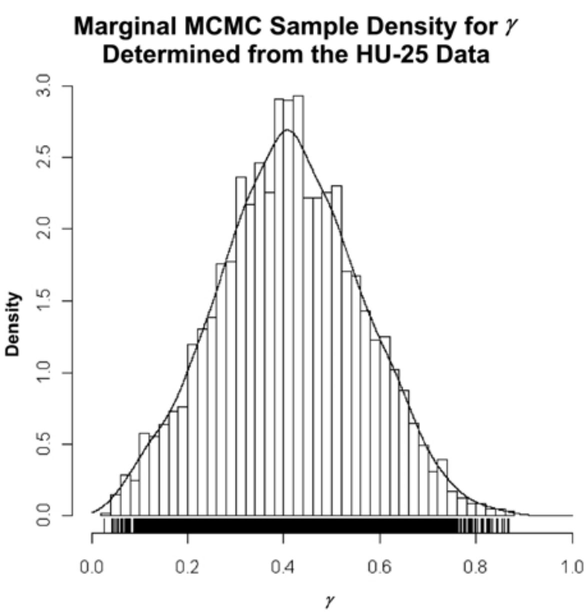

Figure 11 – The marginal MCMC samples for γ are diffuse, but sharper than the statistics in Table 4 suggests.

Figure 11 reveals that there is a 90% probability, based on the HU-25 AC generator data, that $0.16 \le \gamma \le 0.66$. The mode is at $\gamma = 0.41$. Though the samples are fairly diffuse, the sharpness of the peak presents strong evidence that two failure modes are in effect in the HU-25 AC generator data, and that the most likely ratio of failures observed between these two modes is 41%.

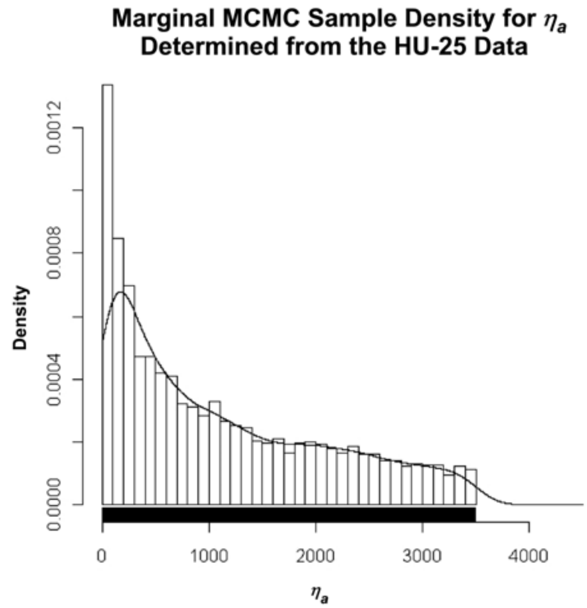

Figure 12 – The marginal MCMC samples and sample density for η_a are relatively small.

Figure 12 suggests that the *critical life* for $Weib_a$ is small. In figure 13, the rather diffuse sampling of figure 12 is much sharper for the failure mode MCMC samples.

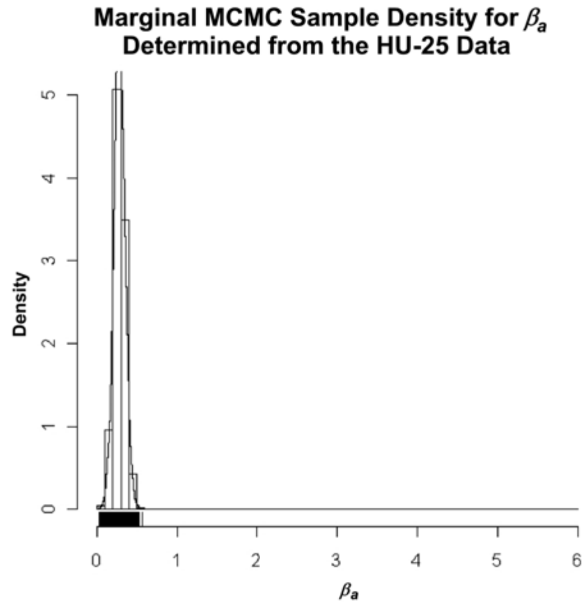

Figure 13 – The marginal MCMC samples for β_a are sharp and indicate a strong *infant mortality* failure mode for *Weib$_a$*.

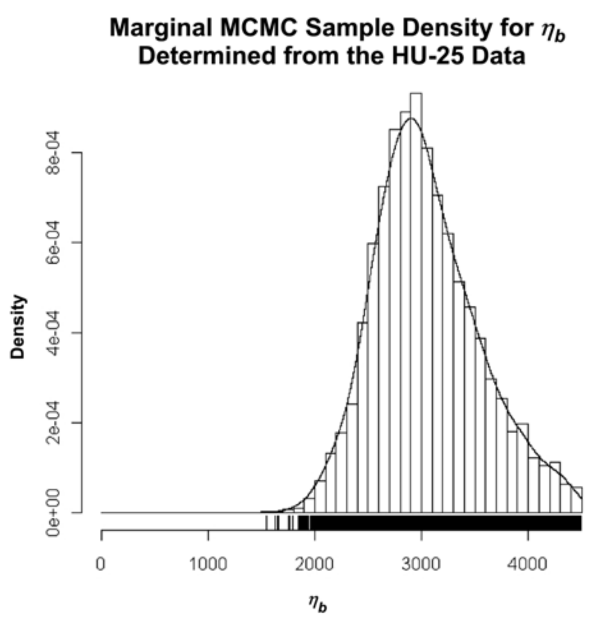

Figure 14 – The marginal MCMC samples and sample density for η_b are relatively sharp compared with figure 12.

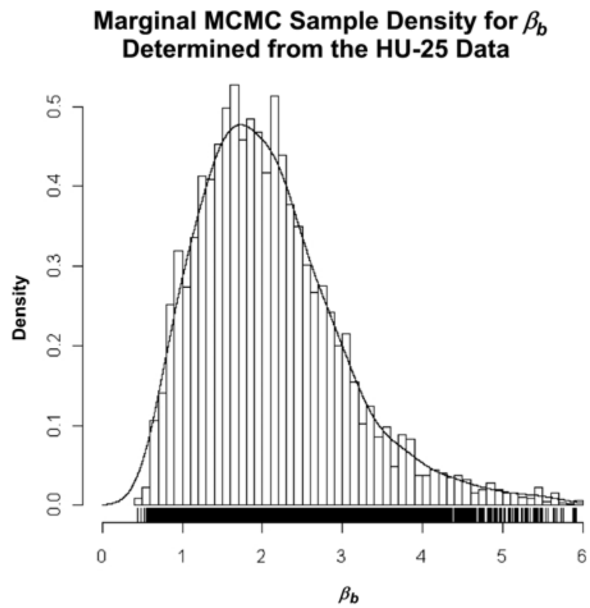

Figure 15 – The marginal MCMC samples and sample density for β_b are not as sharp as those shown in figure 13 for β_a , yet indicate that the second failure mode is very unlikely to be *infant mortality*.

From figure 15, there is a 91.5% probability based on the HU-25 AC generator data that the second failure mode is in *early wearout*. There is very little if any overlap for samples in figure 13 with those in figure 15, additional confirmation of two failure modes.

By examining figures 11-15 together, the procedure described in section 2 applied to the small set of HU-25 AC generator data strongly suggests that two failure modes are in play. One of the failure modes is clearly *infant mortality*, and the other is *early wearout*. As can be seen clearly in figures 13 and 15, there is very little overlap between the MCMC samples of β_a and β_b. The fact that the Markov chains all stabilized, and that the Markov chains for η_a and η_b, and for β_a and β_b, initialized at the same respective values and stabilized in significantly different regions is further evidence of multiple failure modes.

6. CONCLUSIONS

There are a number of important conclusions to be drawn from the discussions in the previous sections.

First and foremost, the model mixing two separate Weibull models can be used to detect and confirm multiple failure modes in a set of data that includes numerous survivor data. The HU-25 AC generator data set was almost 50% survivor data, where the validation data sets contained less than 20% survivor data.

Second, the conditional inferential method presented in section 2 does not require any arbitrary segregation of data, avoiding all of the questions concerning segregation of data, especially for segregation of survivor data.

Third, the conditional inferential method used in conjunction with the mixture model both presented in section 2 can be used to effectively detect and confirm multiple failure modes in failure and survival data, but *only* if they exist. The method specifically fails to achieve stabilization of the Markov chains for some of the parameters if only a single failure mode exists in the data, or a single failure mode dramatically dominates the data. This limits spurious and costly engineering explorations into what could cause phantom failure modes and in how to develop preventative maintenance schedules to improve availability.

Fourth, the method presented in section 2 successfully detected and confirmed multiple failure modes for the HU-25 AC generator from a small set of data which had almost 50% survivors. These failure modes were distinct, and quite different from those found via any data segregation process with reprocessing using a graphical inferential approach.

Future investigations of the method presented in this report remain. The sensitivity of the ability of the method to reliably detect and confirm multiple failure modes with larger proportions of survivor data should be examined by exercising it on validation data sets with more survivors.

Further investigations should be performed into how distinct and different the failure mode distributions must be for the method to reliably detect and confirm multiple failure modes. In this report, the validation data consisted of one *infant mortality* failure mode mixed with one *early wearout* failure mode. Could the method detect and confirm two distinct *early wearout* failure modes, or two distinct *old age* failure modes? This investigation can also be performed with the appropriate validation data sets.

And, lastly, suppose a data set contained more than one failure mode. There is no reason that the method presented in section 2 cannot be expanded to mixtures of more than two models. It may be appropriate to investigate the application of reversible jump MCMC methods to compute probabilities based on the data that the data is represented by one, two, or more failure modes. For today's complex aerospace systems comprised of many complex subsystems, this last investigation may be the most valuable of all.

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BIOGRAPHY

Mark Powell has practiced Systems Engineering for over 35 years in a wide range of technical environments including DoD, NASA, DOE, and commercial. More than 25 of those years have been in the aerospace arena. His roles in these environments have included project manager, engineering manager, chief systems engineer, and research scientist. His academic affiliations have included the University of Idaho, Stevens Institute of Technology, and the University of Houston, Clear Lake. Mr. Powell maintains an active engineering and management consulting practice throughout North America, Europe, and Asia. Beyond consulting, he is sought frequently as a symposium and conference speaker and for training, workshops, and tutorials on various topics in Systems Engineering, Project Management, and Risk Management. Mr. Powell is an active member of AIAA, Sigma Xi, the International Society for Bayesian Analysis, and the International Council on Systems Engineering, where he has served as chair of the Risk Management Working Group, and as Assistant Director for Systems Engineering Processes.