Minimum Cost Test and Verification

New Methods to Put the Assurance Back in Quality

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Introduction

- **Topics for This Evening**
  - **Quality Assurance as an Engineering Specialty**
    - **QA and Test Planning**
    - **QA and Test Results Analysis**
    - **New Methods to Make it Work**
  - **A Real World Example:**
    A Minimum Cost Test Plan for Verifying Reliability of a New Vehicle before Releasing to Market
  - **Closing and Questions**

The Engineering Specialties

- **Reliability, Availability, Maintainability, Safety, Logistics, *Quality Assurance*, etc.**
- **All *Share* a Common Characteristics**
  - ***Probabilistic* in Nature**
  - **Assessment for Decision Making (Requirements' Verification) Requires *Statistical Processing of Data***
  - ***Probabilistic Risk Assessment* (PRA) or Conditional Methods are Preferred for Analysis and Assessment (Required by NASA Directive)**

The Difficulties Using Data

- **Data**
  - **For most Engineering Specialties, *Few* Actual Events**
  - **May have Lots of *Non-Event* Data**
- **Assessments for Decisions always Require a Statistical Process - an *Estimate***
  - **Classical Procedures**
    - **Always *Overconservative***
    - **Rarely Able to Incorporate *Non-Event* Data**
    - ***Cannot* Produce the Uncertainty Model for the Assessment**
  - ***PRA* Procedures**
    - **More *Realistic* than Classical Procedures**
    - **Can Incorporate *Non-Event* Data *Directly***
    - **Can Produce the Uncertainty Model for the Assessment**
    - **However, Uncertainty Model is usually *Analytical Intractable* - not Integrable**

What We Usually Forget

- **Quality Decisions based on Assurance, e.g. QA *Assures* that *Acceptable Risks of not Meeting Requirements* are *not Exceeded* for the System**
  - ***Acceptable Risk* of not Meeting a Requirement is the *Acceptable Probability* of not Meeting a Requirement**
  - ***Values* for *Acceptable Risks* Must be Set**
  - **If the Requirement is an Engineering Specialty Requirement, then Assurance is a *Probability of a Probability!***
- **Test Data Enables an *Assessment***
  - **A *Statistical Estimate* of the Performance Value**
  - **An *Uncertain Value* because of the Statistical Procedure Used**
- **Assurance - *Probability Measures* on the Uncertain Assessment, *Integrals***
- ***SR&QA* - Safety Assurance, Reliability Assurance, and Quality Assurance**
- ***The Role of QA in the Verification/Test Planning Process***

Some Quick Examples

- **Performance Requirements**
  - ***A* - The widget shall be *four meters in length ± 0.1 meters*.**
  - ***B* - The vehicle shall have *95% reliability at 100,000 miles*.**
- **Acceptable Risk Level: < 10% Risk Requirement *not* Satisfied - ≥ 90% Probability that Requirement *is* Satisfied**
- **Test Plans**
  - ***A* - Measure Lengths of 47 Widgets**
  - ***B* - Drive Five Prototype Vehicles to 207,840 Miles**
- **Assessments (*Statistical Estimates*)**
  - ***A* - Mean Length, or Mean and Variance of Length**
  - ***B* - Reliability Assessment given no Prototypes Fail**
- **Assurance: If ≥ 90%, *QA Signs Off***
  - ***A* - Probability Widget Length is between 3.9 and 4.1 Meters**
  - ***B* - Probability Reliability at 100,000 miles is ≥ 95%**

The Difficulties in Test Planning

- **Test Plans are *Inverse Hypothesis Tests***
  - **Based on *Acceptable Risk Level* and Assurance**
  - **Identify *Test Results* that when Analyzed using the Statistical Test Procedure will Produce Needed Assurance**
  - **May have an *Infinite* Set of Test Results that will Work**
- **Classical Statistical Procedures**
  - ***Always Overconservative***
  - **Very Difficult to Use *Non-Event Data* (e.g., no Auto Failures)**
  - **Assurance is Rarely Possible - Uncertainty Model for Assessments not Available, Integrals *Impossible***
- ***PRA* Approaches**
  - ***Realistic* Process Can *Theoretically* Produce Uncertainty Model for Assessments - Assurance is *Theoretically* Possible**
  - **For *Real* Engineering Problems: *PRA* Uncertainty Model is *Analytically Intractable*; Assurance Integrals *Impossible***

The Difficulties in Test Execution

- **Have to Process Data or Test Results using *a Statistical Procedure* - Classical or *PRA***
- ***Unexpected* Data or Test Results**
  - **Failures or Lack of Failures (no events) - when not expected**
  - **Outliers - Unexpected Results**
  - **Missing Data - *Intentional* or *Unintentional***
- **Impossibility of Assurance Integrals**
- **Commonly Used Solutions and their Problems**
  - **Make some *Spurious* Assumptions and Reprocess Data: Making Assumptions - *More* Risk Exposure**
  - ***Forget* Assurance Integral and Use Classical Procedure Assessment *Value* to make Decision: *Overconservative* and *Costly*, Possibly *Arbitrary* Decisions**
  - ***Re-Test* and *Forget* Earlier Results: *You know what happens when you Test until you Get the Answer you want!***

New Methods for Quality Assurance

- ***Monte Carlo*** **Methods are Tried, True, and Time Tested for Computing Probability Integrals**
  - ***Numerical Method*** **of Sampling an Uncertainty (Probability) Model and Calculating Probabilities**
  - ***Could Work for*** **Quality Assurance Quantification, but** ***Must*** **Know the Assessment Uncertainty Model -** ***We Don't***
- **In Mid 1990's,** ***New Methods*** **for Monte Carlo (re)Discovered**
  - **European Biostatisticians Needed to Use** ***PRA*** **for Risky Decisions for Experimental Drug and Surgical Treatments**
  - **Faced Same Problems as in QA -** ***PRA*** **produced Unidentifiable and Analytically Intractable Uncertainty (*Probability*) Models**
  - **Rediscovered** ***Markov Chain Monte Carlo*** **(*MCMC*) Methods**
  - ***MCMC*** **Methods Allow Complete Sampling of Unidentifiable and Analytically Intractable Uncertainty Models**
  - **Numerical Monte Carlo Assurance Integrals Possible with** ***PRA Formulated Test Plans and Executions***

Markov Chain Monte Carlo

- ***Normal Monte Carlo*** **Sampling Provides** ***Independent*** **Random Samples -** ***no*** **Sample to Sample Correlation**
- **Markov Chains Also Provide Random samples, but have** ***Sample to Sample Correlation***
- **For Approximating Numerical Integrals,** ***Who Cares?!!!***
- **Simplest Example of a Markov Chain - the** ***Drunkard's Walk*** **Problem from Physics**
- ***MCMC*** **Unique Capabilities**
  - **Can Sample** ***any*** **Uncertainty Model, Analytically Tractable or not, Proper or Not - with** ***MCMC***, ***PRA Can Work for Assessment and*** **Assurance*****!***
  - **Algorithms are** ***Unbelievably*** **Simple**
  - **Resources for** ***MCMC*** **provided at end of Presentation**

Minimum Cost Test Plan Example

- **Automaker Test Requirements: 95% Reliability at 100,000 miles with 90% *Confidence*; Cheapest Possible Test**
- **Test Cost Function:** $Cost(n,T) = n * \$20,000 + \$1.45 * T$
- **Test Plan Problem: Find Number of Vehicles *n* that will Survive to What Mileage *T* that will Produce ≥ 90% *Confidence* that Vehicle has ≥ 95% Reliability**
- **Basics**
  - **Failures Modeled as *Weibull* Model with β = 3**
  - **To Meet ≥ 95% Reliability, η ≥ 269,141 Miles**

$$R\left(100,000\ miles \mid \beta = 3\right) \geq 0.95 \leq e^{-\left(\frac{100,000}{\eta}\right)^3} \Rightarrow \eta \geq \frac{100,000\ miles}{\left[-\ln\left(0.95\right)\right]^{\frac{1}{3}}} = 269,141\ miles$$

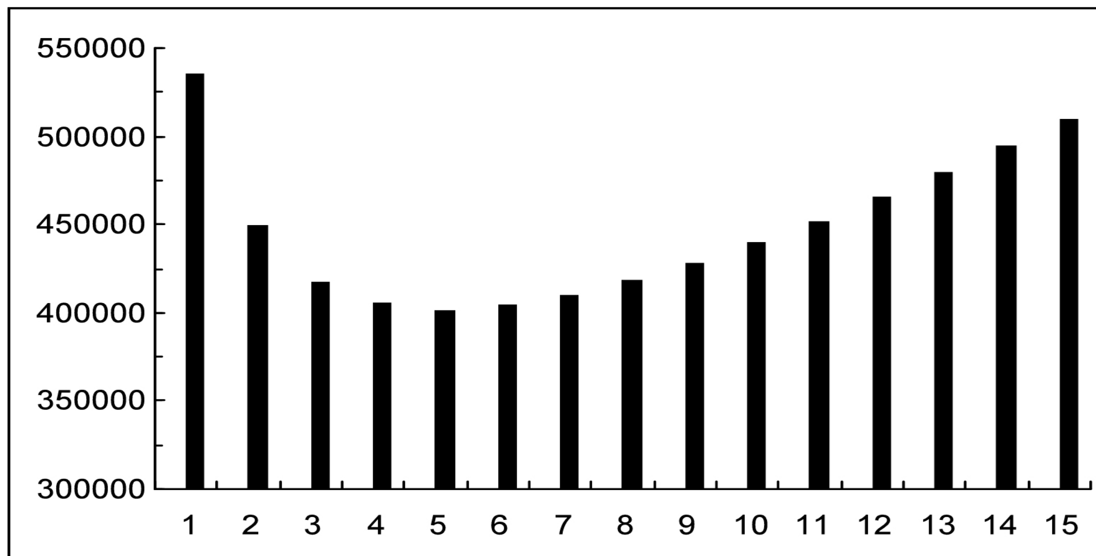
  - **We need *n* and *T* that will produce ≥ 90% *Confidence* from our Test Results (none of the *n* Vehicles fail by Mileage *T*) that η ≥ 269,141 Miles at *Minimum Cost***

The Classical Test Plan

- **Equation from Nelson to Relate *n* and *T* for 90% *Confidence* Level and 95% Reliability**

$$\theta_{L,0.1} = 269{,}141\,miles = \left(\frac{2\sum_{i=1}^{n} T^3}{\chi^2_{0.1,2}}\right)^{\frac{1}{3}} = \left(\frac{2 * n * T^3}{\chi^2_{0.1,2}}\right)^{\frac{1}{3}}$$

- **Dodson Solution Uses Graphical Optimization for Cost:**
  - ***n* = 5**
  - ***T* = 207,840 miles**
  - ***C* = $401,368**



C vs. n

Just a Couple of Minor Problems

- **What is the Probability that if the Vehicles Meet the 95% Reliability Requirement, that the Test will be Successful?**
  - **I.e., Probability that All Five Vehicles Survive to 207,840 miles**
  - **Simple Solution:** $P\left(Pass\,Test \mid \text{Requirement Met}\right) =$

$$P\left(5\,Vehicles\;Survive\;to\;207,840\,miles \mid R\left(100,000\,miles\right) = 0.95\right)$$

$$= \prod_{i=1}^{5} R\left(207,840\,miles \mid \eta = 269,141\,miles\right) = \left(e^{-\left(\frac{207,840}{269,141}\right)^{3}}\right)^{5} = 0.1$$

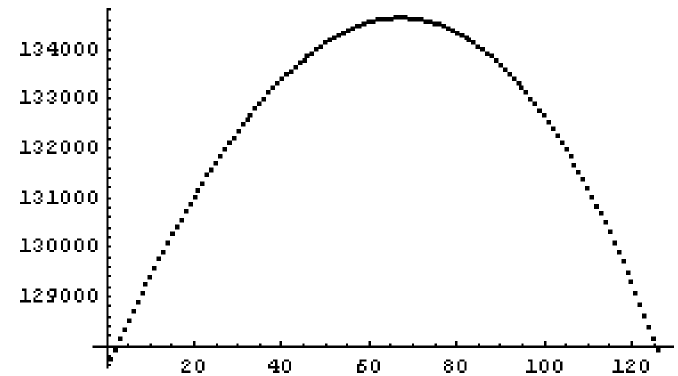
- **What do we do if One or More of the Vehicles *Fail* During the Test?**
  - **Dodson Recommended Replacing it with another Vehicle, Recalculate the Test Duration, and Continue**
  - **What does this do to our Test Plan Cost?**
- ***Is this a Good Test Plan?***

The Second Problem: Unexpected Failures

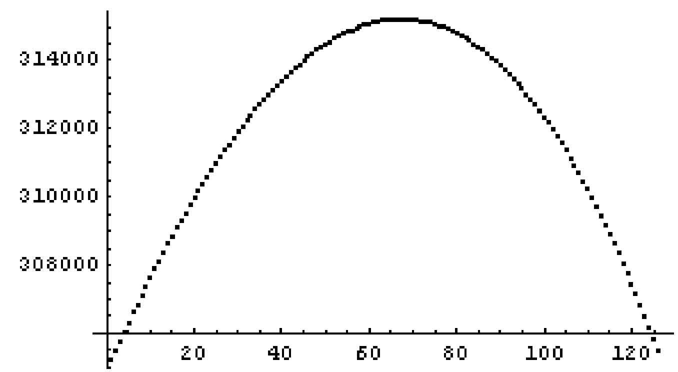
- **Nelson Equation *Generalizes* to Include the One Failure**

$$\theta_{L,0.1} = 269{,}141\,miles = \left(\frac{2 * \left(4 * T^3 + t_f \right)}{\chi^2_{0.1,4}} \right)^{\frac{1}{3}}$$

- **The Classical Statistics *Change!!!!!***
  - **New Maximum Test Duration: $T \leq 134{,}634$ miles**
  - **New Maximum Test Cost: $C \leq \$315{,}213$**
- **New Test Plan**
  - **Break One if all Five Survive to 127,534 miles**
  - ***C* = $285,213**



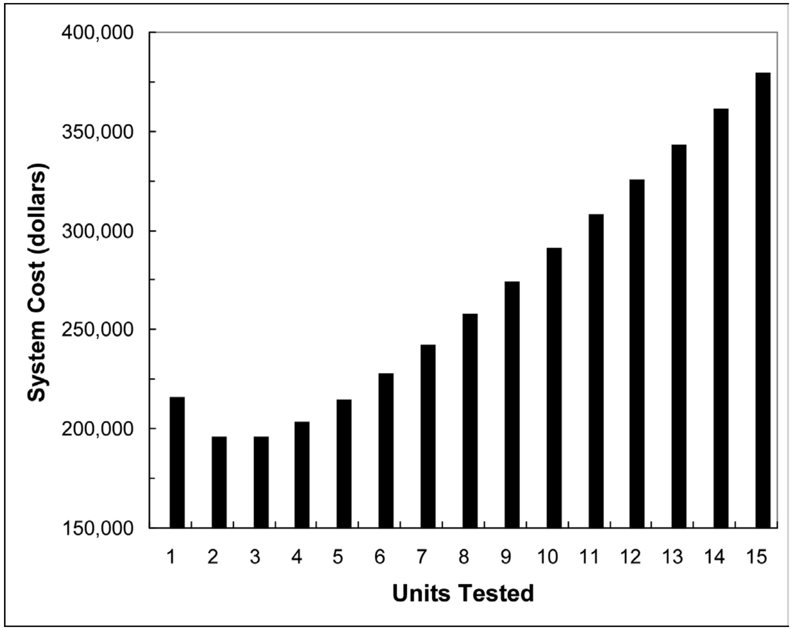
***T* vs. Failure Mileage**



***C* vs. Failure Mileage**

The PRA/MCMC Test Plan

- Use an Ignorance Prior - *Worst Case Scenario*
- Use *MCMC* to find *n* and *T* at 90% Assurance Level; i.e., P($\eta \geq$ 269,241 miles| *n* Survivors to *T* miles)
- Use Dodson Graph to Find Optimum Cost Test Plan
  - *n* = 2; 3
  - *T* = 107,192; 93,641
  - *C* = $195,429; $195,759



$$P\left(Test\ Success \mid R\left(100{,}000\, miles\right) = 0.95\right) = 88.13\%$$

A Little Comparison

	Classical Test Plan	*PRA/MCMC* Test Plan
n	5	3
T	207,840	93,641
C	$401,368	$195,759
P(Success|R=0.95)	10%	88.13%

- **Using an Ignorance Prior and the Classical Test Plan with *PRA* and *MCMC* we Find:**

$$P\left(\text{Requirement Met|Successful Classical Test}\right)$$

$$= P\left(R\left(100{,}000\, miles\right) = 0.95 \mid 5\, Vehicles\ Survive\ to\, 207{,}840\, miles\right) = 0.992$$

- **Automaker Pays *$205,609* for an *Unneeded* (unrequired) *Extra* 9.2% Probability (Assurance) that R(100,000 miles|Successful Test) = 95%**

The PRA/MCMC Plan with a Failure

- **Only 11.87% Probability of a Failure Occurring**
- **Can Use *PRA/MCMC* for a Failure Occurring**
  - **Test Costs *Increase* (this makes sense) Comparable to Original Classical Test Plan**
  - **Probability of Test Success given R(100,000 miles) = 0.95 *Approaches Unity* - Also makes Sense**
  - ***Expected* Test Cost Still *Reasonable***
- **More Test Planning Options may be Considered**

	Zero Failures		One Failure		Zero or One Failure	
Number of Prototypes	**Cost**	**P(Pass Test)**	**Cost**	**P(Pass Test)**	**Expected Cost**	**P(Pass Test)**
1	$215,827.90	0.8813	$713,241.24	0.0548386	$244,966.16	0.9361386
2	$195,428.70	0.8813	$517,393.80	0.080711	$222,441.00	0.962011
3	$195,779.50	0.8813	$465,758.00	0.0840301	$219,280.60	0.9653301
4	$203,363.80	0.8813	$441,570.70	0.0917905	$225,833.58	0.9730905
5	$214,520.80	0.8813	$431,058.35	0.0993902	$236,466.27	0.9806902

Example Synopsis

- **Test Plan Derived Using Classical Procedures Has some Problems**
  - ***Overconservative*** **- extra 9.2% Assurance**
  - ***Too Expensive*** **- More than Twice *PRA/MCMC* Plan Cost**
  - ***Too Unlikely to Succeed*** **- Even if the Requirement is Met**
  - ***Overall Screwy*** **- Consider the Effects of one Failure**
- **Test Plan Using *PRA/MCMC* Procedures Much More Reasonable**
  - ***Realistic*****, even with Ignorance Prior (Worst Case)**
  - ***Likely to Succeed*** **if Requirement Met - 88.13%**
  - ***More Intuitive*****, all the Way Around**
  - **Allows Much More *Flexible* Test Planning and Execution**

Summary

- **Assurance for *QA* is Almost Always Forgotten or Neglected**
  - **With Classical Methods, Impossible to Quantify**
  - **With *PRA* Methods, Analytical Intractability Makes Assurance Calculations Impossible**
- ***PRA* Methods Allow Direct Use of Important and Available *Non-Event Data***
- ***MCMC* Methods Allow Numerical Calculations of Assurance for Analytically Intractable *QA* Solutions from *PRA***
- ***MCMC* Methods Allow Intuitive and Flexible Test Planning and Execution**

PRA/MCMC Methods Put the* Assurance *Back in Quality

Potential Seminar Series Topics

- **Markov Chain Monte Carlo Algorithms**
- **RAM (Reliability, Availability, and Maintainability)**
- **Flight Rule Development**
- **PRA/MCMC Applications**
- **Test Planning**
- **Others by Request**

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Questions

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