Method to Employ Covariate Data in Risk Assessments

IEEE/AIAA

Aerospace Conference

Paper #1653 (13.0204)

10 March 2011

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Introduction

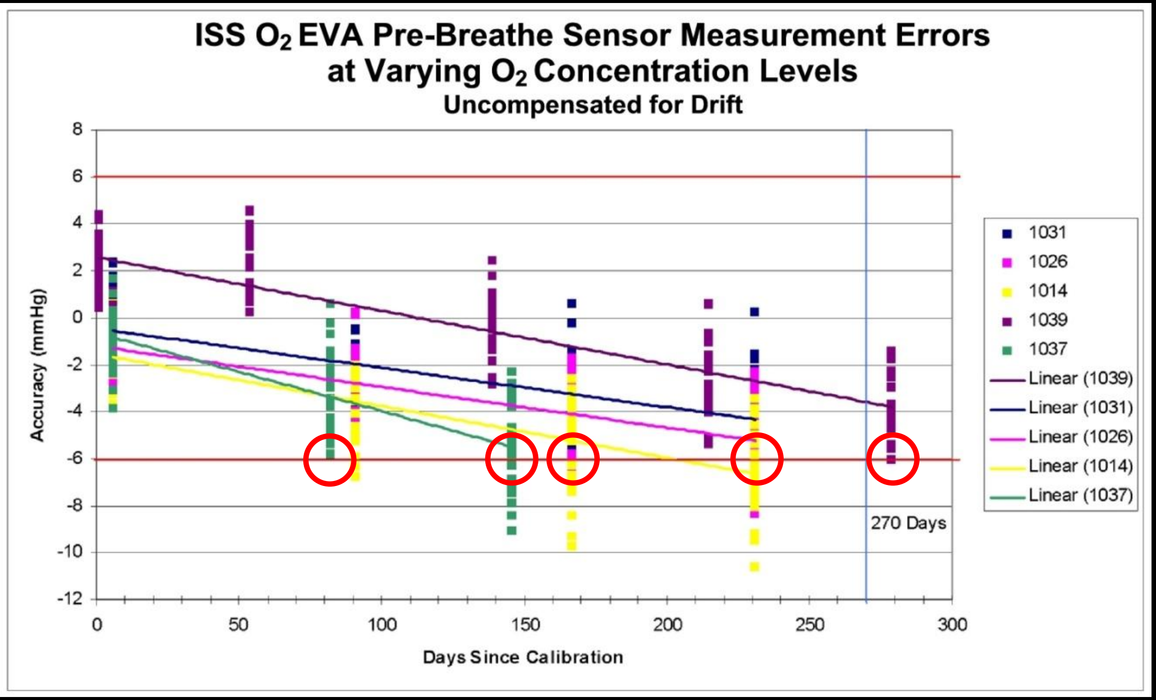
- **Important Decisions for Aerospace Systems Often Based on Formal Risk Assessments**
  - **Usually Low Probability Events**
  - **Usually Very Serious Consequences**
- **Always Best to Consider *All* Information and Data in Performing Risk Assessments**
  - **Data Often Comprised of Events and *Covariates***
  - **Models for Uncertainty Should Incorporate Covariates**
  - **Risk Assessments for Covariate Models Cannot Use Classical Methods – Cannot Process Covariate Information, *So Ignored***
- **Paper #1653 Describes via a NASA Example:**
  - **How to Develop a Model to Incorporate Covariate Data**
  - **Use of Conditional Approaches to Develop a Risk Assessment that Takes Advantage of Covariate Data**
  - **Demonstrates a Much Easier and Very Important Decision**

The Problem

- **On the International Space Station (ISS), The Extra-Vehicular-Activity (EVA) O_2 Sensor Measurements Drifting**
  - **Sensor Accuracy Requirement: $\pm$ 6mmHg for 270 Days post Calibration**
  - **Errors > 6mmHg: Astronaut May Suffer Bends during EVA**
  - **Errors < – 6mmHg: Astronaut May Suffer Oxygen Toxicity**
  - **Either may result in *Death* of Astronauts**
- **NASA Faced with Either**
  - **Halting ISS EVA's Until Sensor Redesign, Testing, and Deployment**
  - **Or, Compensating for the Error Drift to Reduce the Risk**
- **Drift Compensation Results were *not Convincing***

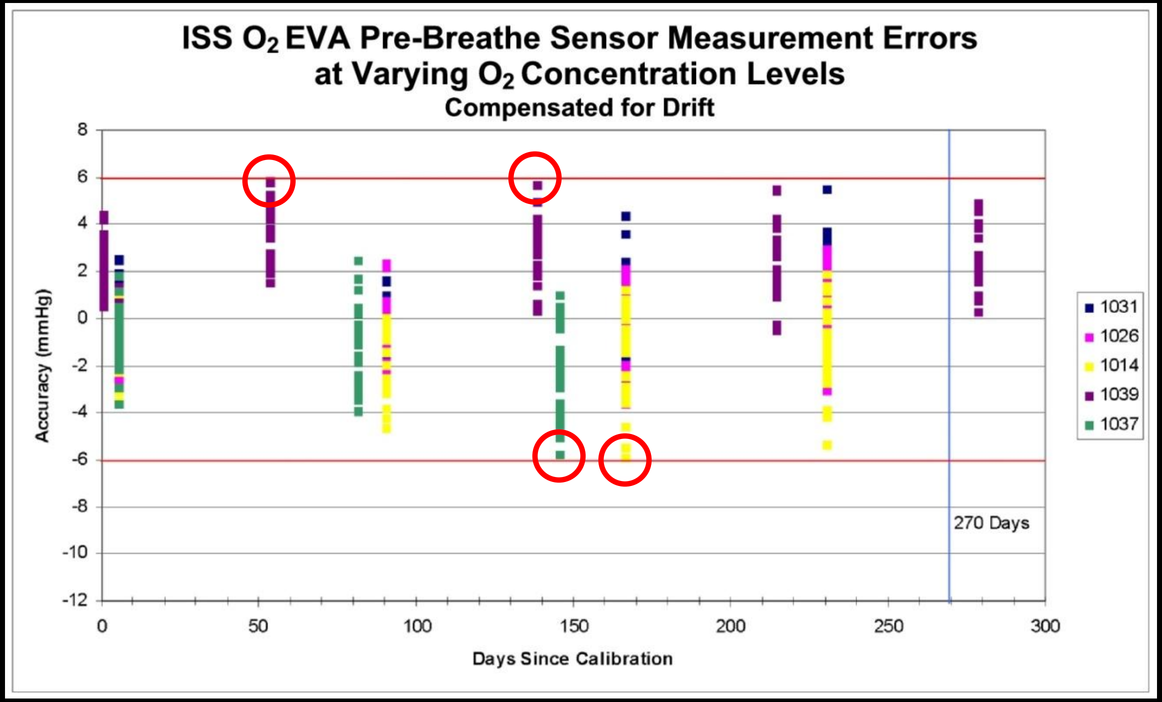
Observed Sensor Errors

- Linear Least Squares Used to Look at Drift for Five Sensors
- All Appeared to Drift in Same Direction, with Similar Rates
- Compensation for Drift Might Reduce the Risk Enough



Compensation Scheme: Use Least Squares on All Data to Estimate Slope and Intercept, and Remove from Sensor Drift Errors

Sensor Errors After Drift Compensation



- **Unacceptable Drift Errors Occur *Even Earlier!***
- **Did the Risk *Actually Increase?***
- **What was the Risk *without* Drift Compen-sation?**
- **No Answers, *No Decision!***

The Important Questions

- **What was the Risk without Drift Compensation during the 270 Days since Calibration?**
- **How Much Did Drift Compensation Reduce the Risk, *if Any*?**

A Method to Obtain the Answers

- **Develop a Sensor Error Drift Model**
  - **Since Linear Least Squares Used to Calculate Drift Compensation Coefficients, Use Normal Model for Errors**
  - **Model Sensor Error Drift as Linear Drift in Error Mean**

$$\mu_s\left(\text{TSC}\right)=\mu_0+\mu' * \text{TSC};\ \ \text{TSC}\equiv \ \text{Time Since Calibration}$$

- **TSC is Covariate with Error Data**
- **Resultant Model for Sensor Errors**

$$pd\left(e_s \mid \mu_0,\mu',\sigma_s,\text{TSC}\right)=\frac{1}{\sqrt{2\pi}\sigma_s}e^{-\left(\frac{1}{2}\right)\left(\frac{e_s-\mu_0-\mu' * \text{TSC}}{\sigma_s}\right)^2}$$

- **Resultant Model is a *Covariate* Normal Model**
- **Develop Joint Uncertainty Distributions for μ_0, μ', and σ_s from Sensor Error Data to Compute Risk Distributions**

Statistical Approach

- ***No*** **Classical Method Exists to Develop Joint Distribution for μ_0, μ', and σ_s for the *Covariate* Model**
- **However, Conditional Inferential Methods Work Very Well for This Type of Problem**
  - **No Questionable Assumptions Necessary**
  - **Will Produce Full Three Dimensional Joint Uncertainty Distribution for μ_0, μ', and σ_s**
  - **In Theory, Risks Before and After Drift Compensation Can be Calculated, Taking into Account Information from the Covariate Data**

Joint Uncertainty Model for μ_0, μ', and σ_s

- **Easy to Formulate Using Conditional Approach**

$$pd\left(\mu_0, \mu', \sigma_s \mid data\right) \propto \prod_{i=1}^{N_{e_s}} \frac{1}{\sigma_s} e^{-\left(\frac{1}{2}\right)\left(\frac{e_{si} - \mu_0 - \mu' * \mathrm{TSC}_i}{\sigma_s}\right)^2} * \left(\frac{1}{\sigma_s}\right)$$

- **Unfortunately, *Not Analytically Integrable***
- **Numerical Methods must be Used – Markov Chain Monte Carlo (*MCMC*)**
  - **Produces Joint Samples of μ_0, μ', and σ_s**
  - **Risk Uncertainty Model can be Derived from Joint Uncertainty Model for μ_0, μ', and σ_s**

Risk Uncertainty Model

- **Calculation of Risk is Also a Function of the Covariate**

$$R\left(|e_s| > e_{\max} \mid \mu_0, \mu', \sigma_s, \mathrm{TSC}\right) = 2 * \Phi\left(-e_{\max} \mid \mu_0 + \mu' * \mathrm{TSC}, \sigma_s\right)$$

$$\text{where } e_{\max} = 6mmHg$$

- **Risk Uncertainty Model is Simple to Formulate**

$$pd\left(R\left(|e_s| > e_{max} \mid \mu_0, \mu', \sigma_s, \mathrm{TSC}\right) \mid data\right)$$

$$\propto R\left(|e_s| > e_{max} \mid \mu_0, \mu', \sigma_s, \mathrm{TSC}\right) * pd\left(\mu_0, \mu', \sigma_s \mid data\right)$$

- **Making the Substitutions Produces a *Messy* Equation**

$$pd\left(R\left(|e_s| > e_{max} \mid \mu_0, \mu', \sigma_s, \mathrm{TSC}\right) \mid data\right)$$

$$\propto 2 * \Phi\left(-e_{\max} \mid \mu_0 + \mu' * \mathrm{TSC}, \sigma_s\right) * \prod_{i=1}^{N_{e_s}} \frac{1}{\sigma_s} e^{-\left(\frac{1}{2}\right)\left(\frac{e_{si} - \mu_0 - \mu' * \mathrm{TSC}_i}{\sigma_s}\right)^2} * \left(\frac{1}{\sigma_s}\right)$$

- **But, That *Messy* Equation Not Really Needed (as will be Seen Later)**

Risk Assurance Calculations

- **Dependence on μ_0, μ', and σ_s is Actually Unimportant – Just Want Covariate Risk Assessment Uncertainty Model Dependent *Only* on the Data**

$$pd\left(R\left(|e_s| > e_{max} \mid \mathrm{TSC}\right) \mid data\right)$$

- **Can Calculate Assurance for Risk by *Integrating* this Risk Uncertainty Model**
- **To Obtain this Covariate Risk Assessment Uncertainty Model, Must Use Marginalization Integrals for μ_0, μ', and σ_s**

$$pd\left(R\left(|e_s| > e_{max} \mid \mathrm{TSC}\right) \mid data\right)$$

$$\propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} R\left(|e_s| > e_{max} \mid \mu_0, \mu', \sigma_s, \mathrm{TSC}\right) * pd\left(\mu_0, \mu', \sigma_s \mid data\right) d\mu_0\, d\mu'\, d\sigma_s$$

$$\propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} \ldots\, d\mu_0\, d\mu'\, d\sigma_s$$

Another Messy Equation, Not Analytically Integrable!

Won't Need It!

Looks Impossible Now

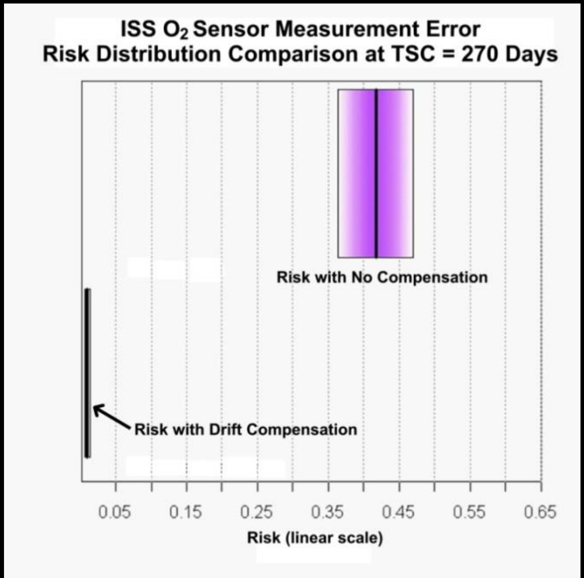
- **Actually, Very Easy to Solve and Obtain Quantitative Risk Assurance – *No Messy Equations Needed!***
- **Suppose Want to Know Assurance *Based on the Data* that Risk of Exceeding e_{max} at TSC = 270 days is Less than 5%**
  - **Use Conditional Covariate Model with MCMC to obtain *M* Joint Samples of μ_0, μ', and σ_s**
  - **Evaluate Risk Calculation Equation at Each Joint Sample of μ_0, μ', and σ_s at TSC = 270 days – Get *M* Samples of Risk of Exceeding e_{max} at TSC = 270 days Based on the Data**
  - **Count Number of Risk Samples < 0.05 and divide by *M***

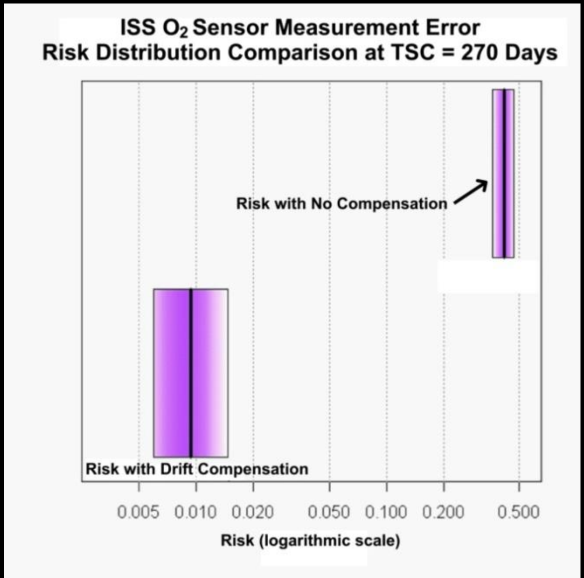
$$P\left(R\left(|e_s| > e_{max} \mid \mathrm{TSC} = 270\right) < 0.05 \mid data\right)$$

$$= \frac{\sum_{i=1}^{M} \begin{bmatrix} 1 \mid 2 * \Phi\left(-e_{max} \mid \mu_{0i} + \mu_i' * 270, \sigma_{si}\right) < 0.05 \\ 0 \mid 2 * \Phi\left(-e_{max} \mid \mu_{0i} + \mu_i' * 270, \sigma_{si}\right) \geq 0.05 \end{bmatrix}}{M}$$

Risk Assessment Results

- **Obtained 10,000 Joint MCMC Samples *of* μ_0, μ', and σ_s for Covariate Data With and Without Drift Compensation**
- **Used to Calculate Risk Samples for both at TSC = 270 days**





- **90% Certain Based on the Data, Risk of Exceeding e_{max} without Drift Compensation within 270 Days Between 36% and 46%**
- **95% Certain Based on the Data, Risk of Exceeding e_{max} with Drift Compensation within 270 Days is less than 1.5%**

Summary of the Process

- **For Risk Assessments with Covariate Data**
  - **Use a Suitable General Uncertainty Model for the Data**
  - **Modify Parameters of this Model as Suitable Functions of the Covariates (Introduces *New* Parameters)**
  - **Use Conditional Approach with MCMC to Sample the Joint Uncertainty Model for the *New* Parameters**
  - **Formulate Risk Calculation Using General Uncertainty Model, Substituting New Parameters as Functions of Covariates for the General Parameters**
  - **Evaluate Risk Calculation Formula at Covariate Values of Interest and at Each Joint MCMC Parameter Sample to Get Samples of Relevant Risk Question**
  - **Calculate Risk Assurances as Needed (Monte Carlo Counting Approach)**

Decide!

Conclusions

- NASA's Decision Suddenly Became *Very Easy*
  - Drift Compensation Actually Reduced Risk More than an Order of Magnitude with High Level of Assurance – *Not Obvious in the Before and After Compensation Data Plots*
  - Based *Solely* on the Data, *No Assumptions*
- Presented One Example of How to Incorporate Covariate Data and Obtain Quantitative Risk Assessment
- Different Problems with Covariates will be Formulated Differently
  - Covariate Models and Covariate Risk Formulae will be Different
  - Conditional Methods with *MCMC* Can be Used to Obtain Useful Answers to Enable Easier Decisions

If You have a Seemingly Impossible Risk Assessment, and Have Data, Contact Me!

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