Method for Investigating Repair/Refurbishment Effectiveness

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Introduction

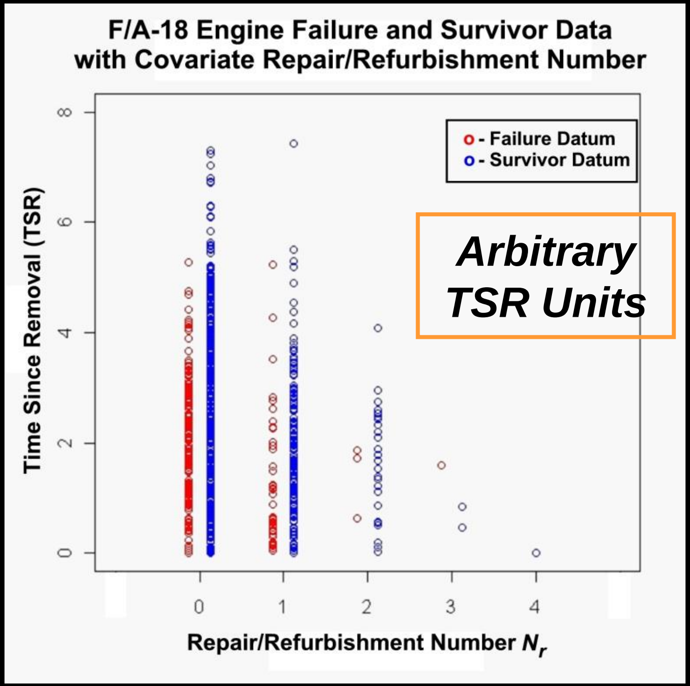
- **Modern Aerospace Systems Often Repaired when they Fail, and Returned to Service**
- **More Often, Repaired or Refurbished as Part of Preventative Maintenance and Returned to Service**
- **Questions:**
  1. ***Do the Repair/Refurbishments Improve, Degrade, or Not Alter Subsequent Reliability of the System?***
  2. ***If the Answer to Question 1 were Known, Could Preventative Maintenance Procedures be Improved?***
- **The F/A-18 E/F Super Hornet, General Electric F414 Low Bypass Gas Turbine Engine is Such a System**

The F/A-18 Engine Data

N_r	0	1	2	3	4	Totals
Failures	193	41	3	1	0	238
Survivors	421	140	29	2	1	593

N_r is the Number of Repair or Refurbishments

- **N_r is Covariate to the Failure and Survivor Data**
- **Over 71% of Data was Survivors**
- **Numbers of Data Fell Off Rapidly with increasing N_r**
- **Fall Off of Number of Failures with increasing N_r Suggests Reliability may Improve with Repeated Repair/Refurbishment**



A Novel Modeling Approach

- Modify a Standard *Weibull* Model to take Advantage of the Information Contained in the Associated Covariate Number of Repair/Refurbishments
  - Modification Should Provide Some *Physical Insight* into the Data (with the Covariate as well)
  - Modification Should Allow Improvements, Degradations, or Stable Reliability as a Function of Increasing N_r
- ***Solution:*** Convert Weibull Parameters into Exponential Functions of N_r Introducing Four New Parameters η_0, β_0, η_c, *and* β_c

$$\eta\left(N_r\right) = \eta_0 * e^{\eta_c * N_r};\ \beta\left(N_r\right) = \beta_0 * e^{\beta_c * N_r}$$

The Covariate Weibull Model

- **Covariate Weibull Parameters:** $\eta(N_r) = \eta_0 * e^{\eta_c * N_r}; \ \beta(N_r) = \beta_0 * e^{\beta_c * N_r}$
- **When N_r = 0, $\eta(N_r)$ = η_0 and $\beta(N_r)$ = β_0 - Standard Weibull**
- **When η_c = 0, *Critical Life* is Constant with Increasing N_r**
- **When β_c = 0, Failure Mode is Constant with Increasing N_r**
- **As $\eta_c \to -\infty$, $\eta(N_r) \to 0$, $\beta_c \to -\infty$, $\beta(N_r) \to 0$**
- **As $\eta_c \to \infty$, $\eta(N_r) \to \infty$, $\beta_c \to \infty$, $\beta(N_r) \to \infty$**

$\eta(N_r)$ and $\beta(N_r)$ Behave Properly!

- **The Resultant Covariate Weibull Model is Somewhat *More Complex* than the Standard Weibull Model**

$$pd(t_f \mid N_r, \eta_0, \beta_0, \eta_c, \beta_c)$$

$$= \left(\frac{\beta_0 * e^{\beta_c * N_r}}{\eta_0 * e^{\eta_c * N_r}} \right) \left(\frac{t_f}{\eta_0 * e^{\eta_c * N_r}} \right)^{\beta_0 * e^{\beta_c * N_r} - 1} e^{-\left(\frac{t_f}{\eta_0 * e^{\eta_c * N_r}} \right)^{\beta_0 * e^{\beta_c * N_r}}}$$

The Impact of the Covariate Weibull Model

- Reliability is now a Function of Time and N_r

$$R(T \mid N_r, \eta_0, \beta_0, \eta_c, \beta_c) = e^{-\left(\frac{T}{\eta_0 * e^{\eta_c * N_r}}\right)^{\beta_0 * e^{\beta_c * N_r}}}$$

- The Uncertainty Model for Reliability as a Function of Time and N_r is Related to the Joint Uncertainty Model for η_0, β_0, η_c, *and* β_c
- No Classical Method Exists to Infer Statistically from the Covariate Data the Joint Uncertainty Model for η_0, β_0, η_c, *and* β_c
- But, Conditional Methods Enable this Inference
  - Without Unnecessary Assumptions
  - Producing the Full Four Dimensional Joint Uncertainty Model for η_0, β_0, η_c, *and* β_c

Joint Uncertainty Model for η_0, β_0, η_c, and β_c

- **Using *Ignorance Priors* for all Parameters, The Joint Uncertainty Model for η_0, β_0, η_c, *and* β_c is:**

$$pd\left(\eta_0,\beta_0,\eta_c,\beta_c \mid data\right)$$

$$\propto \left[\prod_{i=1}^{N_f}\left(\frac{\beta_0 * e^{\beta_c * N_{rfi}}}{\eta_0 * e^{\eta_c * N_{rfi}}}\right)\left(\frac{TSR_{fi}}{\eta_0 * e^{\eta_c * N_{rfi}}}\right)^{\left(\beta_0 * e^{\beta_c * N_{rfi}} - 1\right)} * e^{-\left(\frac{TSR_{fi}}{\eta_0 * e^{\eta_c * N_{rfi}}}\right)^{\beta_0 * e^{\beta_c * N_{rfi}}}}\right] * \left[\prod_{j=1}^{N_s} e^{-\left(\frac{TSR_{sj}}{\eta_0 * e^{\eta_c * N_{rsj}}}\right)^{\beta_0 * e^{\beta_c * N_{rsj}}}}\right] * \left(\frac{1}{\eta_0}\right) * \left(\frac{1}{\beta_0}\right)$$

with TSR_{fi} being the time since repair of the i^{th} failure with covariate number of repairs N_{rfi}, and TSR_{sj} being the time since repair of the j^{th} survivor with covariate number of repairs/refurbishments N_{rsj}.

- **Markov Chain Monte Carlo (*MCMC*) Methods May be Used to Sample this Four Dimensional Joint Uncertainty Model**
- **These Joint Samples of η_0, β_0, η_c, *and* β_c Provide Samples of Covariate Reliability Uncertainty Model**

Covariate Reliability Uncertainty Model

- **The Covariate Reliability Uncertainty Model Resultant from the Covariate Data is *Easy to Formulate***

$$pd\left(R\left(T \mid N_r, \eta_0, \beta_0, \eta_c, \beta_c\right) \mid data\right) = R\left(T \mid N_r, \eta_0, \beta_0, \eta_c, \beta_c\right) * pd\left(\eta_0, \beta_0, \eta_c, \beta_c \mid data\right)$$

- **But, Really only Interested in the Reliability as a Function of Time and N_r based on the Data (not η_0, β_0, η_c, *and* β_c)**
- **Can Obtain via Marginalization Integrals**

$$pd\left(R\left(T \mid N_r\right) \mid data\right)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} pd\left(R\left(T \mid N_r, \eta_0, \beta_0, \eta_c, \beta_c\right) \mid data\right) * d\eta_0 \, d\beta_0 \, d\eta_c \, d\beta_c$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} R\left(T \mid N_r, \eta_0, \beta_0, \eta_c, \beta_c\right) * pd\left(\eta_0, \beta_0, \eta_c, \beta_c \mid data\right) d\eta_0 \, d\beta_0 \, d\eta_c \, d\beta_c$$

This All Looks Impossible!

- **But it isn't!**
  - **With *M* Joint *MCMC* Samples of η_0, β_0, η_c, *and* β_c, these Marginalization Integrals are Easy**
  - **Example: Suppose you wanted to know the *Risk*, based on the Data, that the Reliability at *T* = 2 would not exceed 90% after Two Repair/refurbishments, $N_r = 2$**

- **Simply Evaluate the Covariate Reliability Function at Joint Samples of η_0, β_0, η_c, *and* β_c, with $N_r = 2$ and *T* = 2, to Obtain *M* Reliability Samples**
- **Count the Number of These < 0.9 and Divide by *M***

$$P\left(R\left(T=2 \mid N_r=2\right)<0.9 \mid data\right) = \frac{\sum_{i=1}^{M}\left[\begin{array}{l} 1 \left| \left(e^{-\left(\frac{2}{\eta_{0i} * e^{\eta_{ci} * 2}}\right)^{\beta_{0i} * e^{\beta_{ci} * 2}}} < 0.9 \right) \right. \\ 0 \left| \left(e^{-\left(\frac{2}{\eta_{0i} * e^{\eta_{ci} * 2}}\right)^{\beta_{0i} * e^{\beta_{ci} * 2}}} \geq 0.9 \right) \right. \end{array}\right]}{M}$$

Sounds Good in Theory

- Really Should *Validate* the Concept
  - Pick Representative Values of η_0, β_0, η_c, *and* β_c to use as a Truth Model
  - Generate Set of Failure and Survivor Data for Various N_r
  - Formulate and Run the *MCMC*
  - Compare *MCMC* Joint Samples of η_0, β_0, η_c, *and* β_c based on the Generated Data with True Values

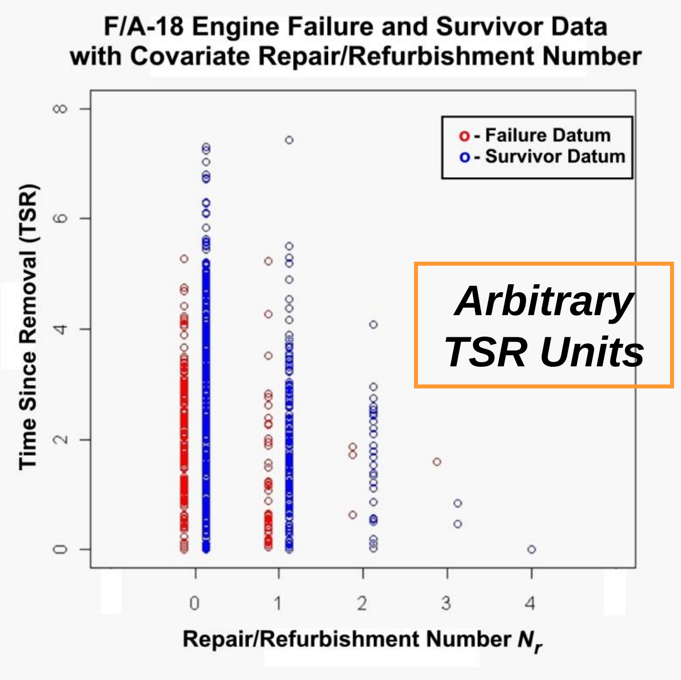
Validation Data Numbers

N_r	0	1	2	3
Failures	794	798	773	792
Survivors	206	202	227	208

Validation Statistics

Parameter	Minimum	Maximum	Mean (True)	σ
η_0	273.9	318.6	295.4 (300)	6.58
β_0	1.38	1.66	1.49 (1.5)	0.039
η_c	-0.162	-0.040	-0.105 (-0.1014)	0.017
β_c	-0.315	-0.214	-0.264 (-0.2747)	0.014

Processing the F/A-18 Engine Data

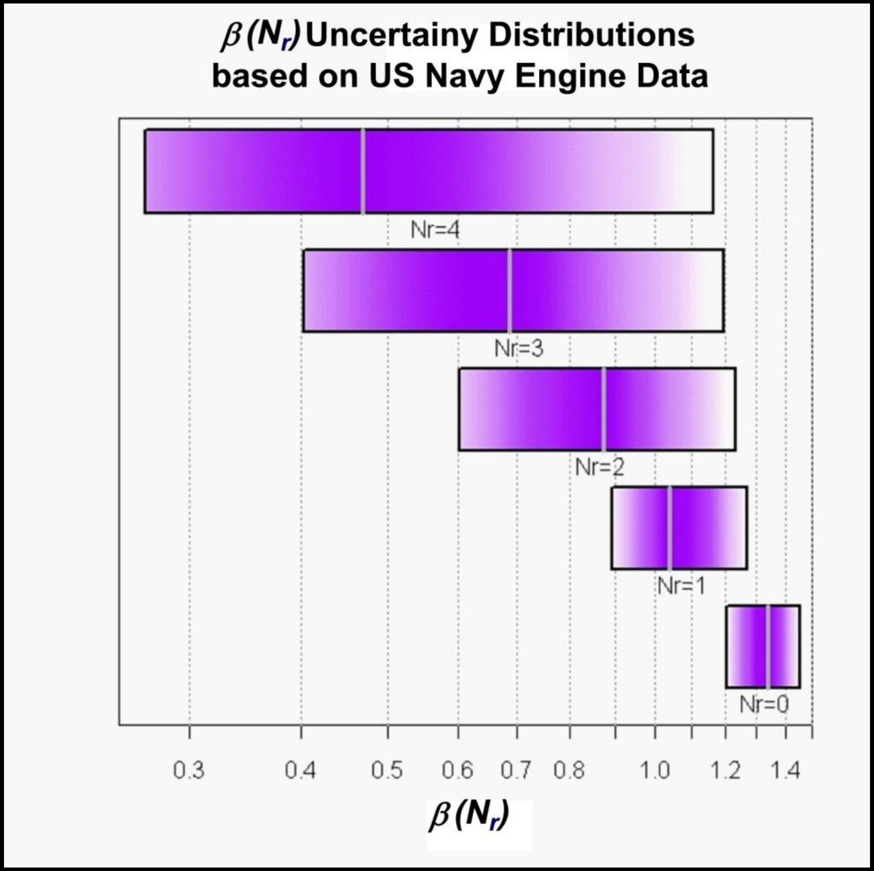


Arbitrary TSR Units

Statistics from 10,000 Joint *MCMC* Samples of η_0, β_0, η_c, *and* β_c from the F/A-18 Engine Data

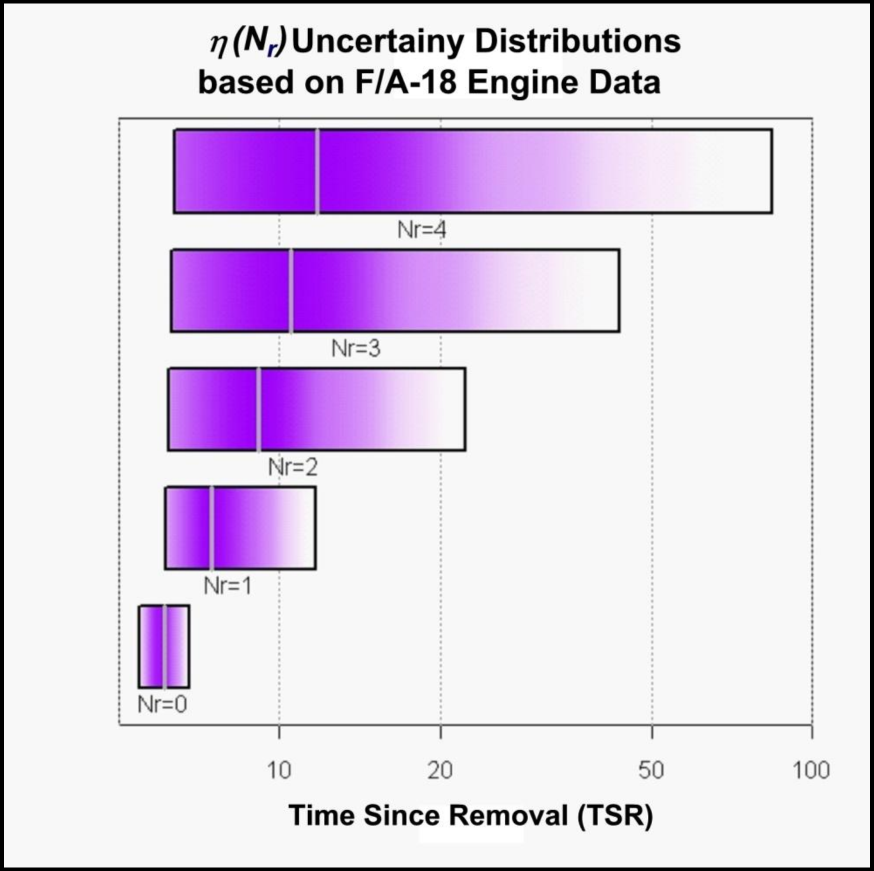
Parameter	Minimum	Maximum	Mean	σ
η_0	4.77	7.47	6.06	0.407
β_0	1.09	1.56	1.33	0.079
η_c	-0.216	1.127	0.303	0.210
β_c	-0.693	0.211	-0.220	0.126

What Does it Mean for F/A-18 Failure Modes?



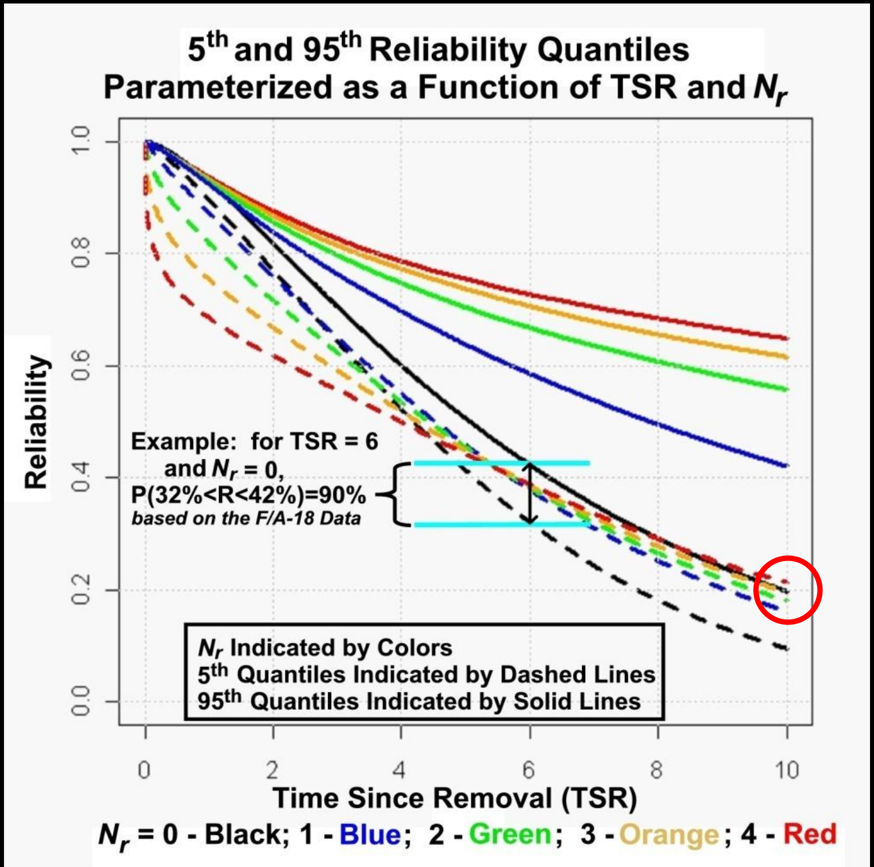
- **F/A-18 Engine Failure Modes Trend More to *Infant Mortality* as N_r Increases**
- **Uncertainty on Failure Mode Increases as N_r Increases**
- **Median F/A-18 Engine Failure Modes Trend More Dramatically to Infant Mortality as N_r Increases**

What Does it Mean for F/A-18 Critical Life?



- **F/A-18 Engine *Critical Life* Increases as N_r Increases**
- **Uncertainty for *Critical Life* Increases as N_r Increases**
- **Median F/A-18 Engine *Critical Life* Increases Exponentially as N_r Increases**
- **More Probability *Meat* at Higher Values**

What Does it Mean for F/A-18 Reliability?

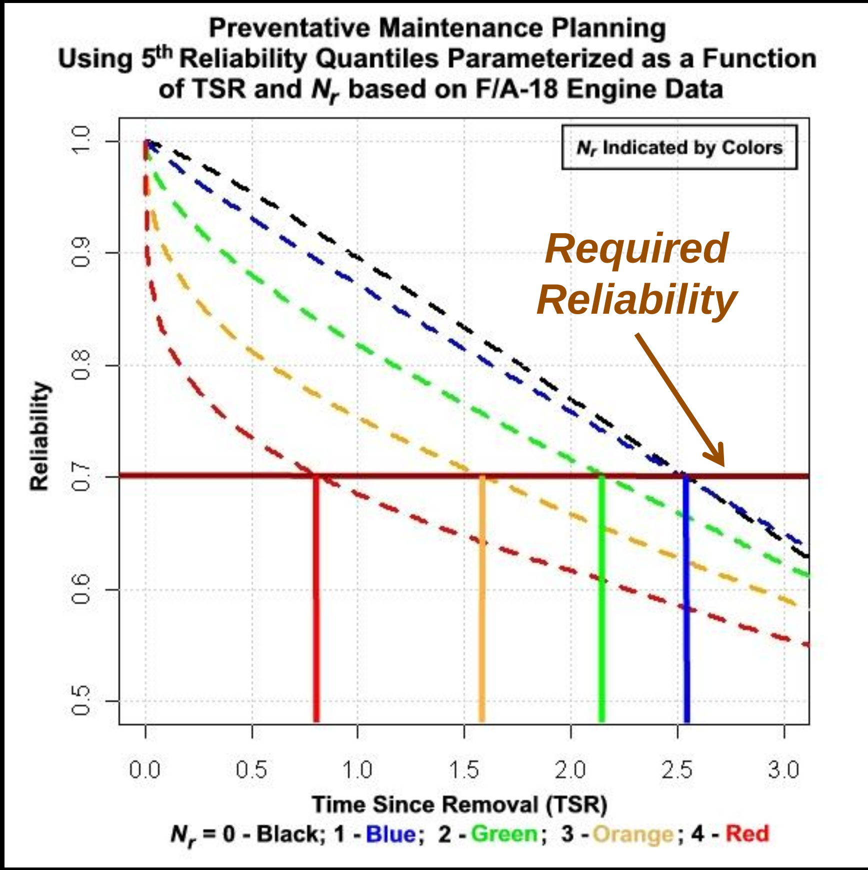


- **95th Quantile Reliabilities Increase as N_r Increases**
- **5th Quantile Reliabilities behave *Non-Intuitively***
  - **At TSR = 10, 95% Probability that Reliability < 20% with N_r = 0, based on the Data**
  - **At TSR = 10, 95% Probability that Reliability > 20% with N_r = 4, based on the Data**

What Could it Mean for F/A-18 PM Planning?

Hypothetical **Example:**

- **Suppose Required Reliability was 70% for Mission Length of 0.5 in TSR Units**
- **Suppose *Maximum Acceptable Risk* of Achieving 70% Reliability for this Mission was 5%**
- **Vertical Lines Show Maximum PM Intervals Based on Data and N_r that Meet the Reliability at Risk Level**



Conclusions

1. Covariate Weibull Model Used with Conditional Inferential and *MCMC* Methods Can Provide Insights into Reliability as a Function of Successive Repair/refurbishments
2. For the F/A-18 Engine Covariate Data, 95% Sure that *Critical Life* Increases with Successive Repair/refurbishments
3. For the F/A-18 Engine Covariate Data, 97% Sure that Failure Mode Moves towards *Infant Mortality* with Successive Repair/refurbishments

More Conclusions

4. Based on Hypothetical Example, Availability, Mission Assurance, and Operational Effectiveness Could be Potentially Improved for the F/A-18 Engine
5. The *MCMC* Samples Obtained via the F/A-18 Engine Data Can be Used to Obtain an Optimal Cost Preventative Maintenance Scheme
6. Non-Intuitive Results thus Obtained Justify Collection of Additional Data to Better Understand What Drives Them
7. This Method may be Used for Any Aerospace System that is Returned to Service after Successive Repair/refurbishments

Final Summary

- A Novel Approach to Investigating Repair/refurbishment Effectiveness Developed, Validated, and Used for the F/A-18 Engine
- Interesting, Potentially Useful, and Some Non-intuitive Results were Obtained for the F/A-18 Engine Covariate Data
- Problems with RCM or PHM that Just Seem Impossible to Solve?

If you Have Data, Contact Me!

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