Cleaner Analysis:Quicker Decisions
Three Examples from Government

Decision Analysis Affinity Group

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Introduction

- **Technical Decisions for Government Projects Often Very Difficult**
  - **Usually, *Lots* of Money Involved**
  - **Lives may be at Stake**
  - **Data may be Sparse**
- **When a Decision Analysis is Presented, Decision Makers Often**
  - **Ask Questions about Assumptions**
  - **Order Re-analysis with Different Assumptions**
  - **Generally Not Satisfied with Point Estimates**
  - **Afraid of Sparse Data**
- **Today will Share Experiences from Three Government Project Decisions for which *Clean Analyses* Led to *Quick Decisions***

Decision Problem #1:
NASA ISS O_2 Sensor Drift

- **On the International Space Station (ISS), The Extra-Vehicular-Activity (EVA) O_2 Sensor Measurements Drifting**
  - **Sensor Accuracy Requirement: $\pm$ 6mmHg for 270 Days post Calibration**
  - **Errors > 6mmHg: Astronaut May Suffer Bends during EVA**

  - **Errors < – 6mmHg: Astronaut May Suffer Oxygen Toxicity**
  - **Either may result in *Death* of Astronauts**
- **NASA Faced with Either**
  - **Halting ISS EVA's Until Sensor Redesign, Testing, and Deployment**
  - **Or, Compensating for the Error Drift to Reduce the Risk**
- **Drift Compensation Results were *not Convincing***

Decision Problem #2:
USCG C130 Cooling Turbine PM

- **Cooling Turbine Provides Cooling and Pressurization to the C130 Crew**
- **Failure in Service**
  - **Loss of Cooling, but More Important, Loss of Cabin Pressurization**
  - **Smoke, Loud, Crew Must Secure**
  - **Mission Compromised**
- **Costs**
  - **Replacement: *$30,000***
  - **Refurbishment: *$500***
- **Most Cost Effective Preventative Maintenance Interval?**

Decision Problem #3: NASA ISS Bone Fracture Risks

- **On-Orbit Astronaut Bone Fractures could have Severe Consequences**
  - **To the Astronaut**
  - **To the Mission**
- **Very Low Probability Event – No Astronaut has Ever Broken a Bone during a Mission in History**
- **Risk Questions**
  - ***What is the Risk of Bone Fracture for Long Mars Missions?***
  - ***How Much will the Risk Increase if International Space Station Missions extend from 180 to 365 Days?***

Approach Used for These Decision Problems

- **Find a Meaningful Decision Discriminator**
  - **Some Physical Quantity or Consequence**
  - **More Importantly, One the Decision Maker *Understands and will Use* to Make the Decision**
- **Gather the Available Data**
- **Use Bayesian Methods**
  - **Use a Most General Model for Data, Express Decision Discriminator in terms of Model Parameters**
  - **Use Non-Informative or Pseudo-Ignorance Priors**
  - **Formulate Joint Posterior Uncertainty Distribution for Model Parameters**
- **Sample Joint Posterior Uncertainty Distribution using *Markov Chain Monte Carlo* (*MCMC*) Methods**

An Aside:
Markov Chain Monte Carlo

- **Just Like Ordinary Monte Carlo, Except for Sampling Approach**
  - **Ordinary Monte Carlo Uses *Built-in* Samplers for *Recognizable* Models**
    - **Usually Only Univariate Samplers Available**
    - **Possible Exception, Multivariate Normal**
  - ***MCMC* uses a Markov Chain to Sample a Density Function**
    - **Any Density Function – not Restricted to Recognized Built-in Models, *and Any Dimension!***
    - **Any Combination of Discrete and Continuous, One-sided, Two-sided, and Interval Random Variables**
- **All that is Required to Use *MCMC* is an Analytical Expression for the Density Function**

Continuing with the Approach

- **Use *MCMC* Samples for the Joint Posterior Uncertainty Distribution for Parameters to Develop Samples of the Uncertainty Distribution for the Decision Discriminator, based on the Data**
- **Parameterize if Necessary**
- **Present *Uncertainty Distributions for the Decision Discriminator* for All Alternatives**

Notes:
The Available Data

- **Most of these Tough Decision Problems Have Few *if Any* Event Data**
  - **Risks of Financial Loss Should be Small**
  - **Risks of Failure Should be Very Small**
  - **Risks of Loss of Life Should be Tiny**
- **May have Plenty of *Censored* Data**
  - **Observations that Event or Loss has Not Occurred**
  - **Classical Statistical Approaches Almost Always Ignore**
  - **Resulting Bayesian Posterior Formulations Almost Always Analytically Intractable**
- **May have Outliers – *Or, Maybe Not***
  - **May be Just one of those Rare Events**
  - **Should *never* Ignore Outliers**

Notes:
Bayesian Methods

- **Decision Theory/Analysis has Long Historical Basis using Bayesian Methods**
- **Select the Most General Model Possible for Data**
  - **One-sided Data:** ***Weibull***
  - **Two-sided Data:** ***Non-central t***
  - **Interval Data:** ***Beta***

Avoids Some Assumptions

- **Use of Non-Informative, or Jeffreys', or Ignorance, or Reference Priors Obviates Questionable Assumptions**
  - **Usually Produces Analytically Intractable Joint Posterior**
  - **Forced to Use** ***MCMC***
  - **To Achieve Markov Chain Stability, Sometimes Must Wisely Truncate the Ignorance Prior –** ***Pseudo-Ignorance Prior***
- **Bounds Results Consistent with Information Theory**

Notes:
Using MCMC Joint Samples to Obtain Decision Discriminator Uncertainty Model Samples based on the Data

- **Fairly Simple Process: Evaluate Decision Discriminator at Joint MCMC Samples of Parameters**
- **What this Accomplishes**

$$pd\left(D\,|\,data\right) \propto \underset{\substack{parameter\\domains}}{\iiint} D\left(params\right) * pd\left(params\,|\,data\right) dparams$$

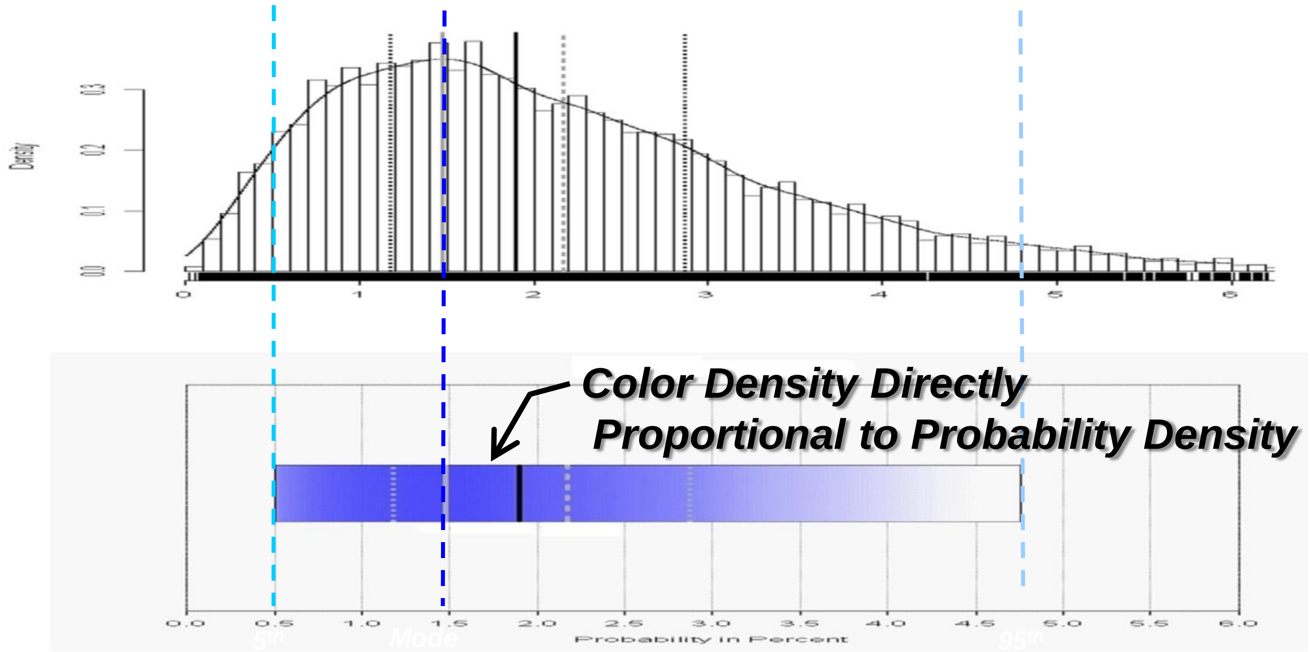
  - **Performs the Required Marginalization Integrals**
  - **Produces Samples of the Uncertainty Model for the Decision Discriminator**

If Needed, Parameterize

- **For Continuous Alternatives: Parameterize Decision Discriminator Uncertainty Distributions as Function of Alternative**
- **For Data with Covariates: Parameterize Decision Discriminator Uncertainty Distributions as Function of Covariates**
- **Simple, Merely Requires CPU Time**
- **Avoids a lot of Decision Maker *What if* Questions, as well as a lot of Analysis Repeats**

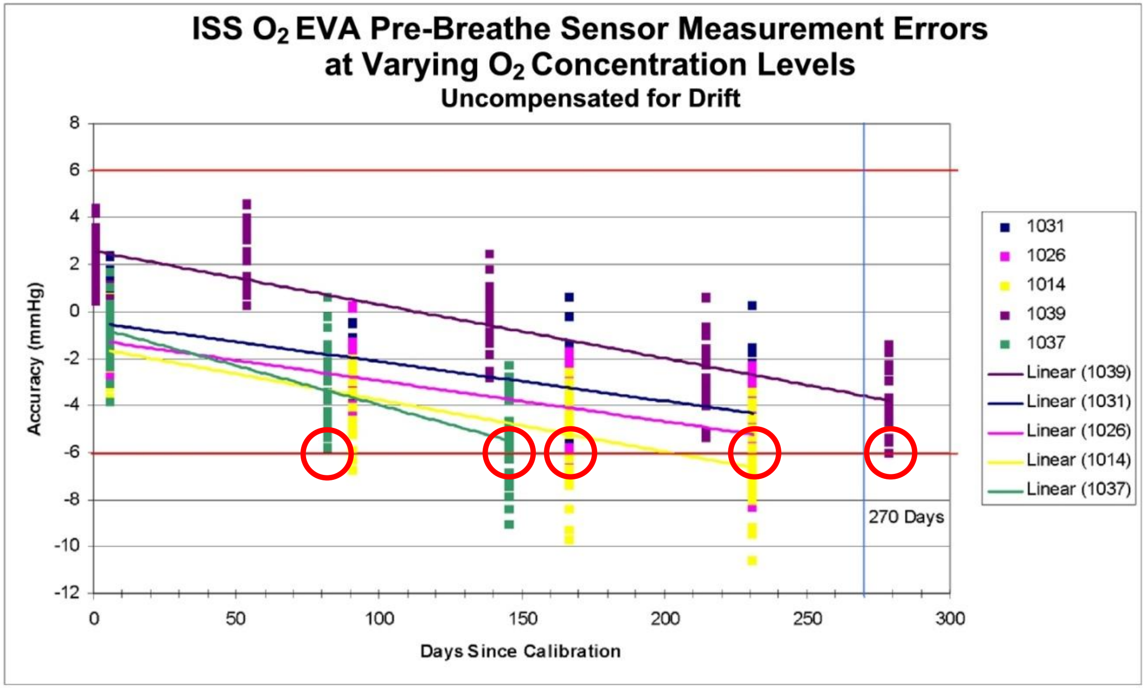
Alternative Distribution Presentations

- **For Discrete Alternatives, Modified Bar Charts Work Well for Risk Comparisons**



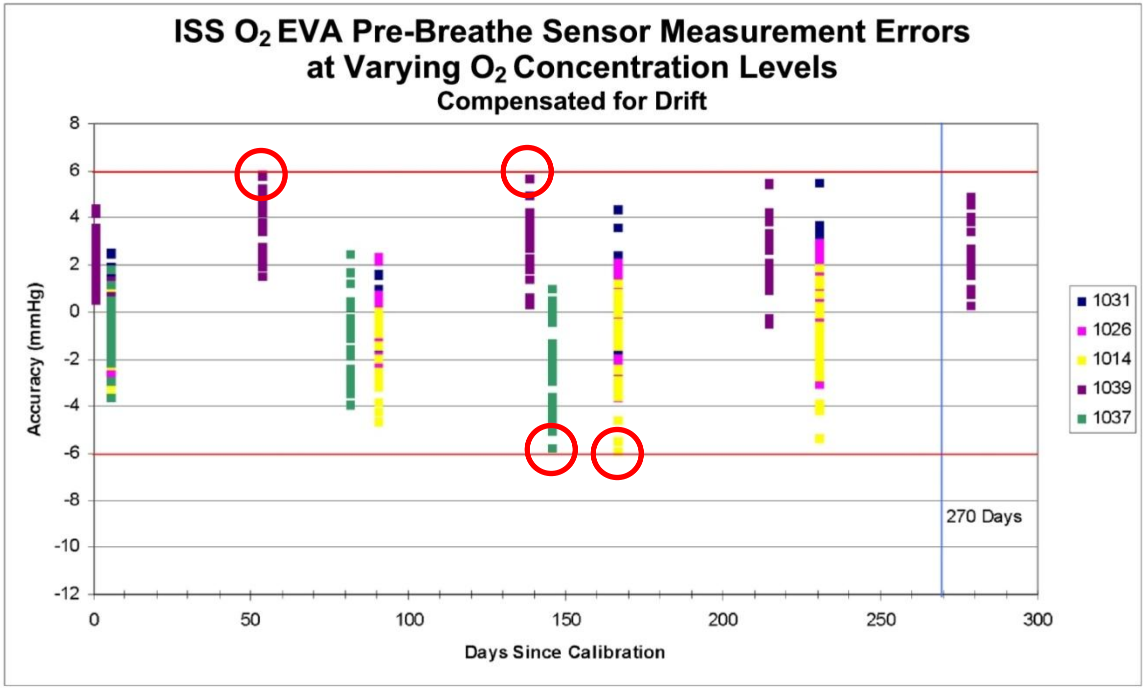
Now, Decision Problem #1
Observed Sensor Errors

- **Linear Least Squares Used to Look at Drift for Five Sensors**
- **All Appeared to Drift in Same Direction, with Similar Rates**
- **Compensation for Drift Might Reduce the Risk Enough**



Compensation Scheme: Use Least Squares on All Data to Estimate Slope and Intercept, and Remove from Sensor Measurements

Sensor Errors After Drift Compensation



- **Unacceptable Drift Errors Occur *Even Earlier!***
- **Did the Risk *Actually Increase?***
- **What was the Risk *without* Drift Compen-sation?**
- **No Answers, *No Decision!***

Decision Analysis

- **Decision Discriminator: Risk of Exceeding e_{max} (±6mmHg) at TSC = 270 days** $R\left(|e_s| > e_{max} \mid 270,\, \mu_0, \mu', \sigma_s\right) = 2 * \Phi\left(-e_{\max} \mid \mu_0 + \mu' * 270, \sigma_s\right)$
- **Data: Preceding Slides, Before and After Compensation**
- **Bayesian Approach**
  - **Used Normal Model with Covariate for TSC since Linear Regression was Used to Compute Drift Correction Parameters**
  - **Joint Posterior with Ignorance Priors *NOT* Analytically Tractable, Used *MCMC* Sampling**
- **Decision Discriminator Uncertainty Model Transform**

$$pd\left(R\left(|e_s| > e_{max} \mid 270\right) \mid data\right)$$

$$\propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\ldots \right] d\mu_0\, d\mu'\, d\sigma_s$$

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Not A… …able!

Won't Need It!

- **Use Modified Bar Charts before and after Compensation**

Another Aside:

Decision Discriminator Uncertainty

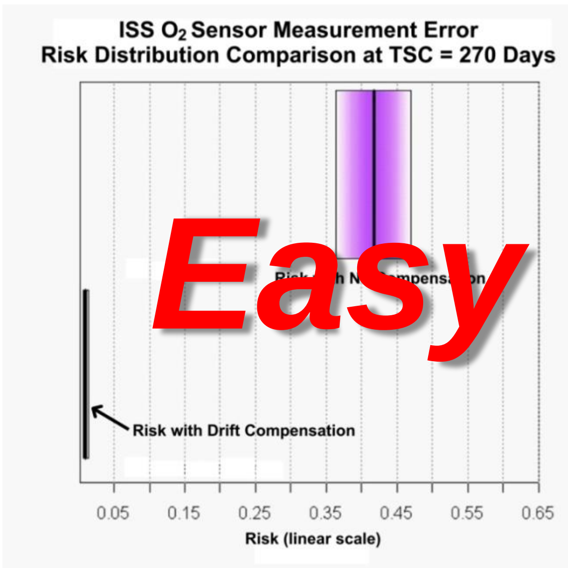
- **Obtain Samples by Simply Evaluating Decision Discriminator Equation at Joint Samples of Posterior**
- **Suppose Want to Know Assurance *Based on the Data* that Risk of Exceeding e_{max} at TSC = 270 days is Less than 5%**
  - **Have *M* Joint Posterior Samples from MCMC**
  - **Evaluate Decision Discriminator Equation at Each Joint Sample at TSC = 270 days – Get *M* Samples of Risk of Exceeding e_{max} at TSC = 270 days Based on the Data (for Modified Bar Charts)**
  - **Count Number of Risk Samples < 0.05 and divide by *M***

$$P\left(R\left(\left|e_s\right| > e_{max} \mid \mathrm{TSC} = 270\right) < 0.05 \mid data\right)$$

$$= \frac{\sum_{i=1}^{M} \begin{bmatrix} 1 \mid 2 * \Phi\left(-e_{max} \mid \mu_{0i} + \mu_i' * 270, \sigma_{si}\right) < 0.05 \\ 0 \mid 2 * \Phi\left(-e_{max} \mid \mu_{0i} + \mu_i' * 270, \sigma_{si}\right) \geq 0.05 \end{bmatrix}}{M}$$

Risk Assessment Results

- **Obtained 10,000 Joint MCMC Samples *of* μ_0, μ', and σ_s for Covariate Data With and Without Drift Compensation**
- **Used to Obtain Risk Samples for both at TSC = 270 days**



Easy Decision!

- **90% Certain Based on the Data, Risk of Exceeding e_{max} without Drift Compensation within 270 Days Between 36% and 46%**
- **95% Certain Based on the Data, Risk of Exceeding e_{max} with Drift Compensation within 270 Days is less than 1.5%**

Decision Problem #2

The C130 PM Problem

- **60:1 Cost Ratio, Replace:Maintain**
- **Only Had Five Failure Data: 463, 538, 1652, 1673, and 2462 flight hours**
- **Only Had One Survivor Datum: 96 flight hours**
- **What PM Interval to Select?**
- **USCG Decision Makers *Paralyzed***

Decision Analysis

- **Decision Discriminator: CS_{tpm} – Cost Savings per Flight Hour in performing Preventative Maintenance at the Interval of t_{pm} flight hours over Allowing Failures in Service**

$$CS_{tpm} = \left(\frac{C_{rep}}{\eta}\right)\gamma\left(\frac{\beta-1}{\beta},\left(\frac{t_{pm}}{\eta}\right)^{\beta}\right) - \left(\frac{C_{pm}}{t_{pm}}\right) * e^{-\left(\frac{t_{pm}}{\eta}\right)^{\beta}}$$

- **Data: Preceding Slides, 5 Failures Events, One Survivor**
- **Bayesian Approach**
  - **Used Weibull Model**
  - **Posterior with Ignorance Priors *NOT* Analytically Tractable, Use *MCMC* Sampling**
- **Decision Discriminator Uncertainty Model Transform**

$pd(CS_{tpm}$ …

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Not A… …able!

Won't Need It!

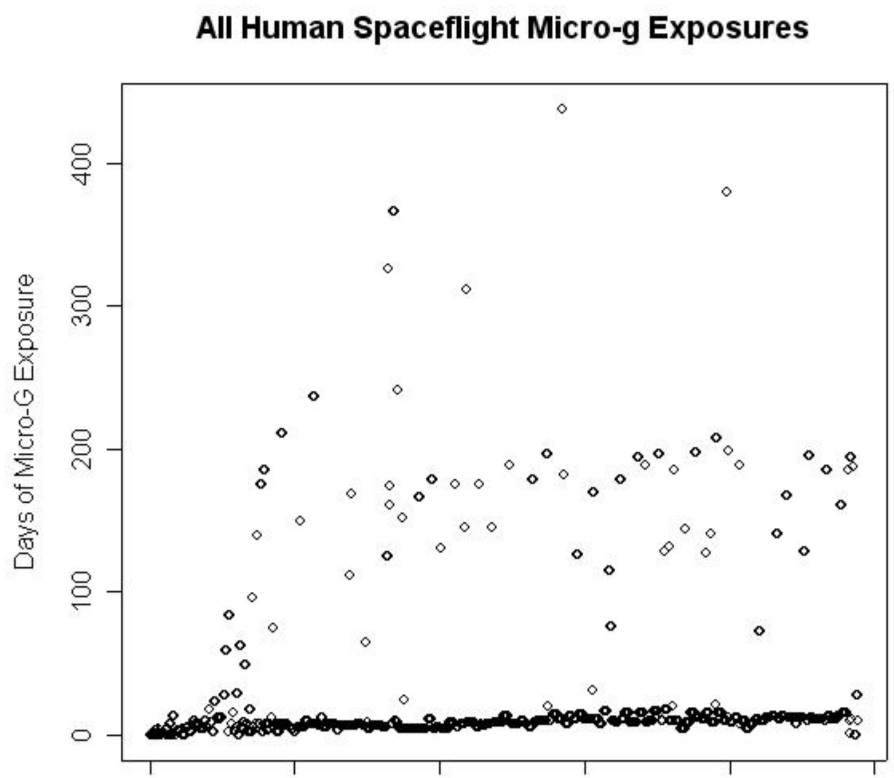
- **Use Parameterization as a Function of t_{pm}**

Cost Savings Risks Using a PM Interval - Parameterized

- **Full Distributions Per Flight Hour Based Solely on The Data, Per Bird – CS_{tpm}**
- **Obtained by Evaluating CS_{tpm} at the Joint Posterior MCMC Samples Parameterized as a function of PM Interval in flight hours**
- **Plotted Only 5th, Most Likely, and 95th percentile Cost Savings Risks**
- **At t_{pm} = 250 hours, 95% Certain, based on the data, that USCG can *SAVE* at least $17 per flight hour per bird**

Decision Problem #3
Available NASA Bone Fracture Data

- **977 Astronaut Missions of Varying Lengths (as of May 2005)**
- **No Events Observed**
  - **No Bones Broken**
  - **Did Observe 977 Mission Lengths without a Broken Bone**



Decision Analysis

- **Decision Discriminator: *Risk of Bone Fracture*** $R_{T_M} = 1 - e^{-\left(\frac{T_M}{\eta}\right)^{\beta}}$
- **Data: Preceding Slides, 977 Censored Data**
- **Bayesian Approach**
  - **Used Weibull Model**
  - **Posterior with Ignorance Priors *NOT* Analytically Tractable, Use *MCMC* Sampling**
- **Decision Discriminator Uncertainty Model Transform**

$pd\left(R_{T_M}\right. \quad \ldots \quad d\eta\, d\beta$

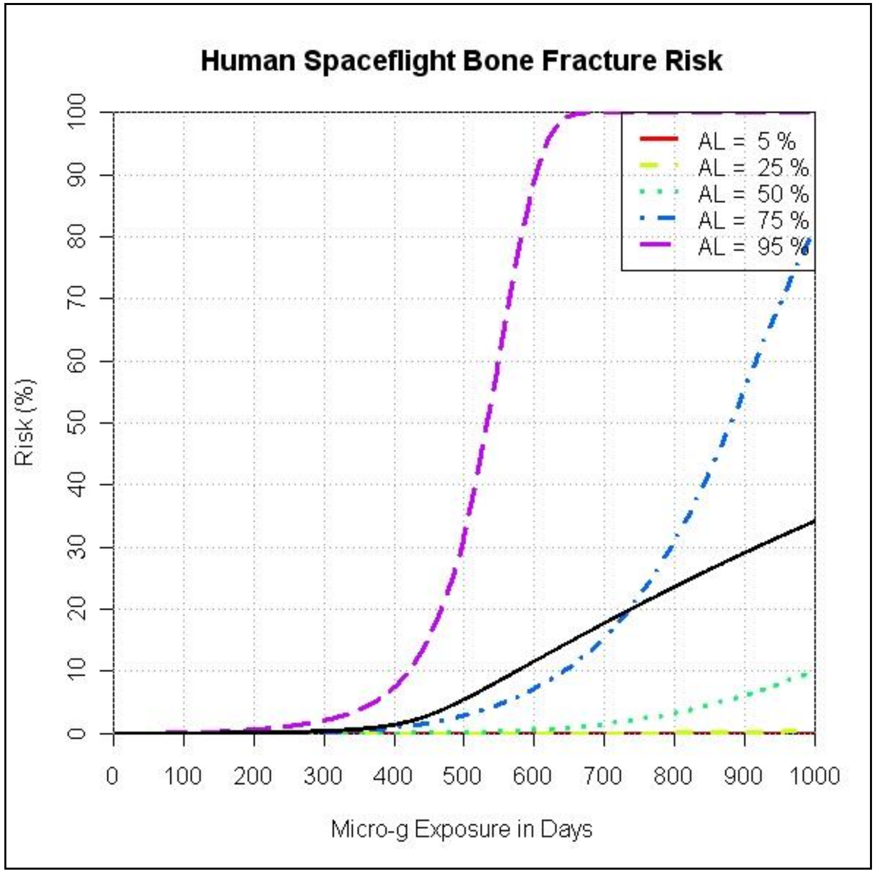
A Ve… …tion,
Not A… …able!

Won't Need It!

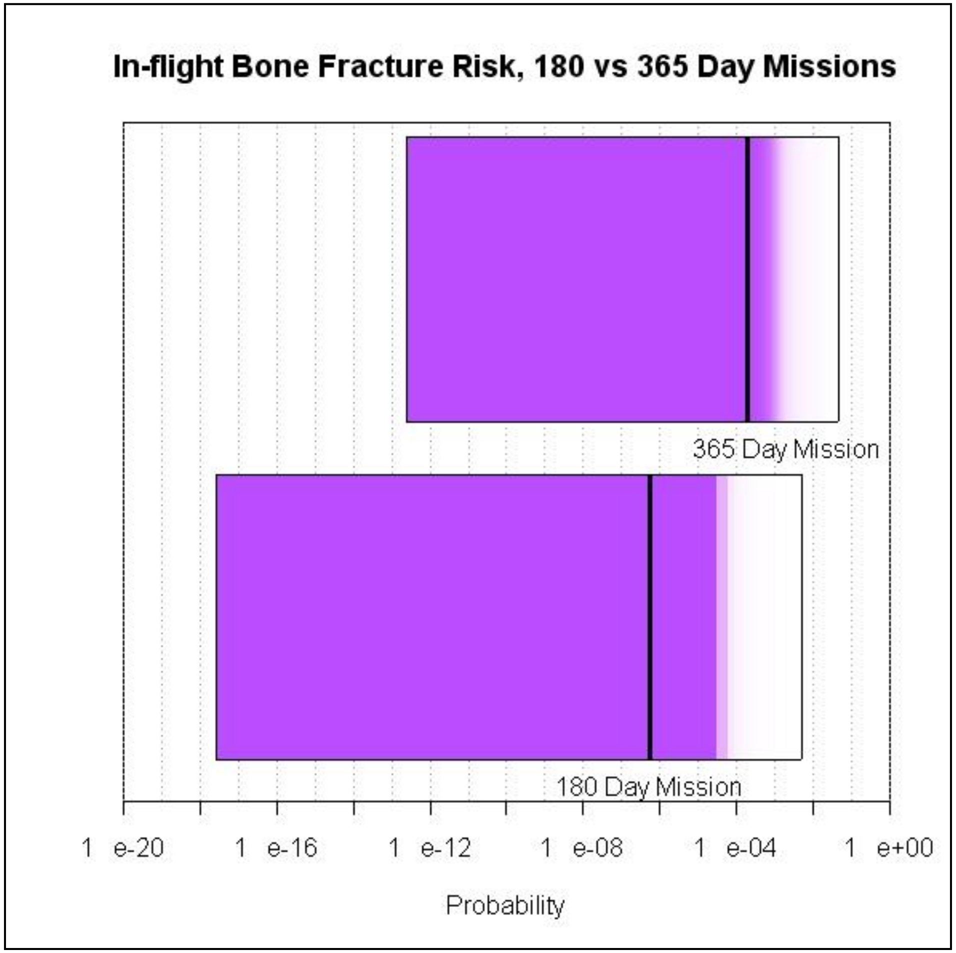
- **Use Parameterization and Modified Barcharts**

Risk Results Parameterized for Mission Duration

- **Risk Uncertainty Distribution As a Function of Mission Duration Obtained by Evaluating Risk Equation at MCMC Samples**
- **Plotted Various Assurance Levels (5, 25, 50, 75, 95%)**
- **For Mars Missions of 270 Days – We Can be 95% Certain that Risk of fracture during the Mission is < 3%, *Based on the Information Available***



The ISS Mission Extension Question



Summary

- **The Decision Analyses Used were *Clean***
  - **Selected Meaningful and Useful Decision Discriminators**
  - **Using Ignorance or Pseudo-Ignorance Priors Limited Use of Questionable Assumptions**
  - **Used Parameterizations**
  - **Presented Uncertainty Distributions for Decision Discriminators for All Alternatives, based on the Data**
- **Decisions Made *Almost Immediately* for All Three Examples**
  - **Decision Makers were Comfortable Deciding**
  - **Saved Money in All Cases**

Conclusion

- **Have Published Papers for these 3 Examples, Contact me and I will Share**
- **Available to Help with *Tough* Decision Analysis Problems**
- **Or, I Can Teach Your Folks How to Perform Clean Decision Analysis and Achieve Quick Decision**
- ***Link with me*: http://www.linkedin.com/in/attwatermarkpowell**

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